XXVI International Physics Olympiad

Canberra, ACT
Australia

Theoretical Competition

July 7, 1995

Time Allowed: 5 Hours

READ THIS FIRST

Permitted Materials: Non Programable Calculators

Instructions:

1. **Use only the pen provided**

2. **Use only the marked side of the paper**

3. **Begin each problem on a separate sheet**

4. **Write at the top of every sheet:**

   - **The number of the problem**
   - **The number of the sheet in your solution for each problem**
   - **The total number of sheets in your solution to the problem.**

Example (for Problem 1 with 3 sheets): 1 1/3
1 2/3
1 3/3

Do not staple your sheets. They will be clipped together for you at the end of the examination.

Question 1

Gravitational Red Shift and the Measurement of Stellar Mass

(a) (3 marks)
A photon of frequency f possesses an effective inertial mass m determined by its energy. We may assume that it has a gravitational mass equal to this inertial mass. Accordingly, a photon emitted at the surface of a star will lose energy when it escapes from the star's gravitational field. Show that the frequency shift Δf of a photon when it escapes from the surface of the star to infinity is given by

$$\frac{\Delta f}{f} \simeq -\frac{GM}{Rc^2}$$

for $\Delta f << f$ where
G = gravitational constant
R = radius of the star
c = velocity of light
M = mass of the star.

Thus, the red shift of a known spectral line measured a long way from the star can be used to measure the ratio M/R. Knowledge of R will allow the mass of the star to be determined.

(b) (12 marks)
An unmanned spacecraft is launched in an experiment to measure both the mass M and radius R of a star in our galaxy. As the spacecraft approaches its objective radially, photons emitted from He^+ ions on the surface of the star are monitored via resonance excitation of a beam of He^+ ions in a test chamber inside the spacecraft. Resonance absorption occurs only if the He^+ ions are given a velocity towards the star to allow exactly for the red shifts. The velocity ($v = ßc$) of the He^+ ions in the spacecraft relative to the star at absorption resonance is measured as a function of the distance d from the (nearest) surface of the star. The experimental data are displayed in the accompanying table. Fully utilize the data to determine graphically the mass M and radius R of the star. There is no need to estimate the uncertainties in your answer.

Data for Resonance Condition

Velocity parameter	$\beta = v/c$ (10^{-5})	3.352	3.279	3.195	3.077	2.955
Distance from surface of star	d $(10^8 m)$	38.90	19.98	13.32	8.99	6.67

(c) In order to determine R and M in such an experiment, it is usual to consider the frequency correction due to the recoil of the emitting atom. [Thermal motion causes emission lines to be broadened without displacing distribution maxima, and we may therefore assume that all thermal effects have been taken into account.]

(i) (4 marks)
Let E be the energy difference between two atomic energy levels, with the atom at rest in each case. Assume that the atom decays at rest, producing a photon and a recoiling atom. Obtain the relativistic expression for the energy hf of a photon emitted in terms of E and the initial rest mass m_o of the atom.

(ii) (1 mark)

Hence, make a numerical estimate of the relativistic frequency shift $\left(\frac{\Delta f}{f}\right)_{\text{recoil}}$ for the case of He^+ ions.

Your answer should turn out to be much smaller than the gravitational red shift obtained in part (b).

Data:

Velocity of light	c	=	3.0×10^8 m/s
Rest energy of He	$m_o c^2$	=	4×938 (MeV)
Bohr energy	E_n	=	$-\frac{13.6\ Z^2}{n^2}$ (eV)
Gravitational constant	G	=	6.7×10^{-11} Nm^2 kg^{-2}.