Question 2

Sound Propagation

Introduction

The speed of propagation of sound in the ocean varies with depth, temperature and salinity. Figure 1(a) below shows the variation of sound speed c with depth z for a case where a minimum speed value c_o occurs midway between the ocean surface and the sea bed. Note that for convenience z = 0 at the depth of this sound speed minimum, $z = z_s$ at the surface and $z = -z_b$ at the sea bed. Above z = 0, c is given by;

$$c = c_o + bz$$

Below z = 0, c is given by;

$$c = c_o - bz$$

In each case $b = \left|\frac{dc}{dz}\right|$, that is, b is the magnitude of the sound speed gradient with depth; b is assumed constant.

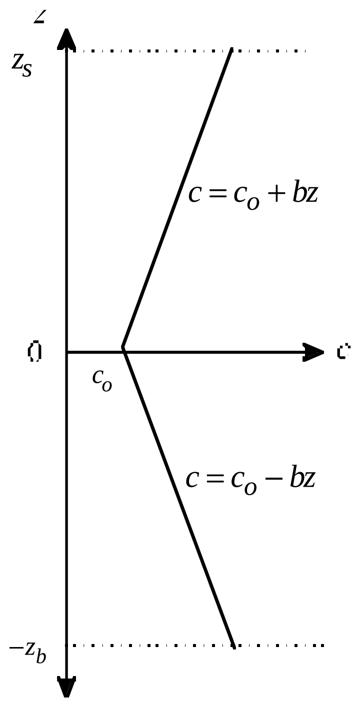

Figure 1 (a)

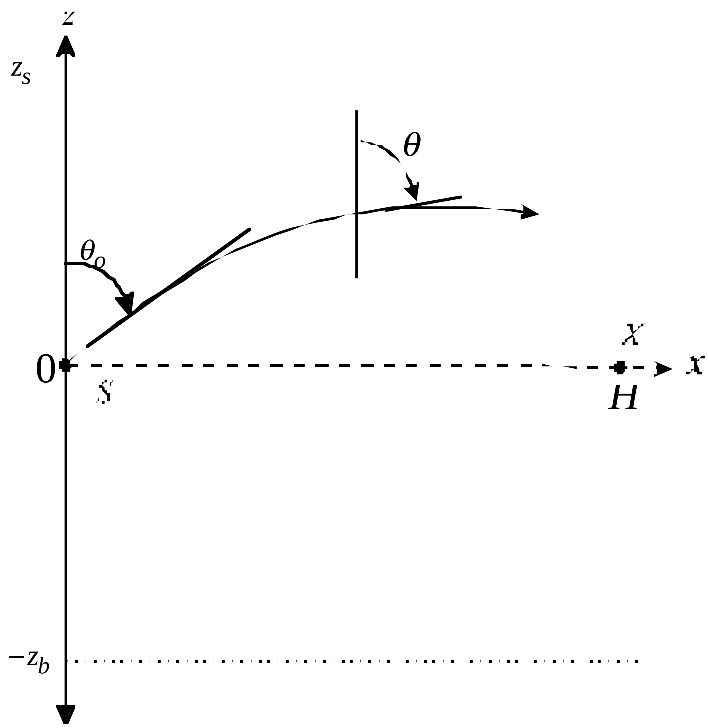

Figure 1 (b)

Figure 1(b) shows a section of the *z*-*x* plane through the ocean, where *x* is a horizontal direction. At all points along the *z*-*x* section the sound speed profile $c(z)$ is as shown in figure 1(a). At the position $z = 0$, $x = 0$, a sound source *S* is located. Part of the output from this source is described by a sound ray emerging from *S* with initial angle θ_o as shown. Because of the variation of sound speed with *z*, the ray will be refracted, leading to varying values along the trajectory of the ray.

(a) (6 marks)
Show that the initial trajectory of the ray leaving the source *S* and constrained to the *z*-*x* plane is an arc of a circle with radius *R* where:

$$R = \frac{c_o}{b \sin \theta_o} \quad \text{for} \quad 0 \le \theta_0 < \frac{\pi}{2}$$

(b) (3 marks)
Derive an expression involving z_S, c_O and b to give the smallest value of the angle θ_o for upwardly directed rays which can be transmitted without the sound wave reflecting from the sea surface.

(c) (4 marks)
Figure 1(b) shows the position of a sound receiver *H* which is located at the position $z = 0$, $x = X$. Derive an expression involving b, X and c_O to give the series of values of angle θ_o required for the sound ray emerging from *S* to reach the receiver *H*. Assume that z_S and z_b are sufficiently large to remove the possibility of reflection from sea surface or sea bed.

(d) (2 marks)
Calculate the smallest four values of θ_o for refracted rays from *S* to reach *H* when;

$X = 10{,}000$ m
$c_O = 1{,}500$ m/s
$b = 0.02000\text{s}^{-1}$

(e) (5 marks)
Derive an expression to give the time taken for sound to travel from *S* to *H* following the ray path associated with the **smallest** value of angle θ_o, as determined in part (c). Calculate the value of this transit time for the conditions given in part (d). The following result may be of assistance:

$$\int \frac{dx}{\sin x} = \ln \tan\left(\frac{x}{2}\right)$$

Calculate the time taken for the direct ray to travel from *S* to *H* along $z = 0$. Which of the two rays will arrive first, the ray for which $\theta_o = \frac{\pi}{2}$, or the ray with the smallest value of θ_o as calculated for part (d)?