**Task 1**

1a.
$\Delta\theta_{\text{nominal}}=5'=0.08°$

| $\Delta\theta_{\text{nominal}}$ (degree) | 0.08 |
|---|---|

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1b.



Reflected Beam

Incident Beam

$\theta_0$

a

Optical Axis
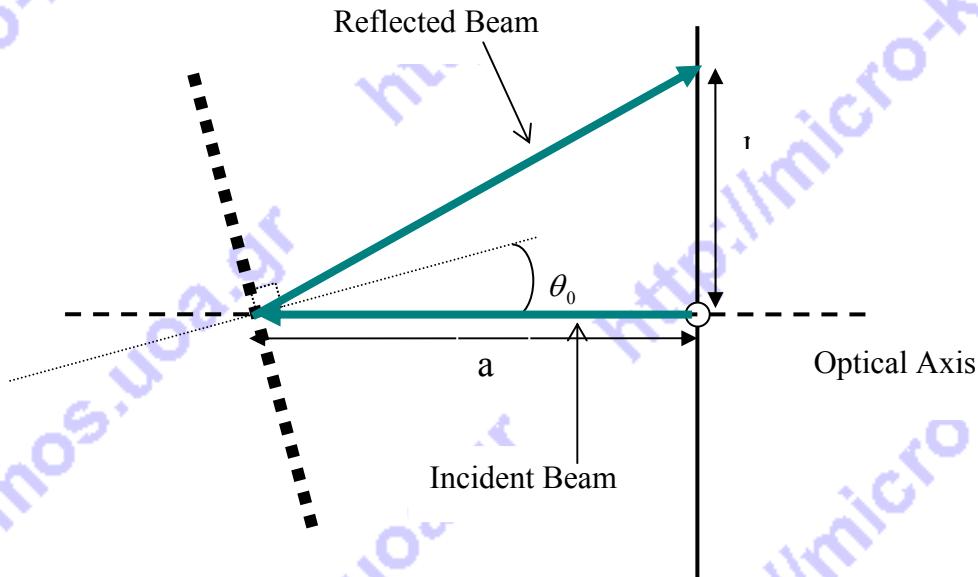
If "a" is the distance between card and the grating and "r" is the distance between the hole and the light spot so we have

$$\Delta f(x_1, x_2,...) = \sqrt{\left(\frac{\partial f}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2}\Delta x_2\right)^2 + ...}$$

$$\tan(2\theta_0) = \frac{r}{a}, \; If \; \theta_0 << 1 \Rightarrow \theta_0 = \frac{r}{2a} \Rightarrow \Delta\theta_0 = \sqrt{\left(\frac{\Delta r}{2a}\right)^2 + \left(\frac{r\,\Delta a}{2a^2}\right)^2}$$

We want $\theta_0$ to be zero i.e. $r = 0 \Rightarrow \Delta\theta_0 = \frac{\Delta r}{2a}$

$$\Delta r = 1mm, \; a = (70 \pm 1)mm \Rightarrow \theta_0 = \frac{\Delta r}{2a} \; rad = 0.007 \; rad = 0.4°$$

| $\Delta\theta_0$ | 0.4° |
|---|---|
| $\theta$ range of visible light (degree) | 13° ≤ θ ≤ 26° |

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1c.

| $R_{\min}^{(0)}$ | (21.6±0.1) kΩ |
|---|---|
| $\Delta\varphi_0$ | 5′ = 0.08° |
| $R_{\min}^{(1)}$ | R=(192±1) kΩ |

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$\Delta\varphi_0 = 5'$ because
$\theta = 5' \Rightarrow R = (21.9 \pm 0.1)$ kΩ
$\theta = -5' \Rightarrow R = (21.9 \pm 0.1)$ kΩ

1d.

Table 1d. The measured parameters

| θ (degree) | $R_{glass}$(MΩ) | $\Delta R_{glass}$(MΩ) | $R_{film}$(MΩ) | $\Delta R_{film}$(MΩ) |
|---|---|---|---|---|
| 15.00 | 3.77 | 0.03 | 183 | 3 |
| 15.50 | 2.58 | 0.02 | 132 | 2 |
| 16.00 | 1.88 | 0.01 | 87 | 1 |
| 16.50 | 1.19 | 0.01 | 51.5 | 0.5 |
| 17.00 | 0.89 | 0.01 | 33.4 | 0.3 |
| 17.50 | 0.68 | 0.01 | 19.4 | 0.1 |
| 18.00 | 0.486 | 0.005 | 10.4 | 0.1 |
| 18.50 | 0.365 | 0.005 | 5.40 | 0.03 |
| 19.00 | 0.274 | 0.003 | 2.66 | 0.02 |
| 19.50 | 0.225 | 0.002 | 1.42 | 0.01 |
| 20.00 | 0.200 | 0.002 | 0.880 | 0.005 |
| 20.50 | 0.227 | 0.002 | 0.822 | 0.005 |
| 21.00 | 0.368 | 0.003 | 1.123 | 0.007 |
| 21.50 | 0.600 | 0.005 | 1.61 | 0.01 |
| 22.00 | 0.775 | 0.005 | 1.85 | 0.01 |
| 22.50 | 0.83 | 0.01 | 1.87 | 0.01 |
| 23.00 | 0.88 | 0.01 | 1.93 | 0.02 |
| 23.50 | 1.01 | 0.01 | 2.14 | 0.02 |
| 24.00 | 1.21 | 0.01 | 2.58 | 0.02 |
| 24.50 | 1.54 | 0.01 | 3.27 | 0.02 |
| 25.00 | 1.91 | 0.01 | 4.13 | 0.02 |
| 16.25 | 1.38 | 0.01 | 66.5 | 0.5 |
| 16.75 | 1.00 | 0.01 | 40.0 | 0.3 |
| 17.25 | 0.72 | 0.01 | 23.4 | 0.2 |
| 17.75 | 0.535 | 0.005 | 12.8 | 0.1 |
| 18.25 | 0.391 | 0.003 | 6.83 | 0.05 |
| 18.75 | 0.293 | 0.003 | 3.46 | 0.02 |
| 19.25 | 0.235 | 0.003 | 1.76 | 0.01 |
| 19.75 | 0.195 | 0.002 | 0.988 | 0.005 |
| 20.25 | 0.201 | 0.002 | 0.776 | 0.005 |
| 20.75 | 0.273 | 0.003 | 0.89 | 0.01 |

| Range | Zero error |
|---|---|
| 200 MΩ | 0.9 MΩ |

1e.

In θ=-20˚ => $R_{glass}$= (132± 2) kΩ , $R_{film}$= (518±5) kΩ

| $\theta$ | $T_{film}$ | $\theta$ | $T_{film}$ |
|---|---|---|---|
| $\theta = -20˚$ | 0.255 | 19.25 | 0.134 |
| | | 19.50 | 0.158 |
| | | 19.75 | 0.197 |
| | | 20.00 | 0.227 |
| | | 20.25 | 0.259 |
| | | 20.50 | 0.276 |
| | | 20.75 | 0.307 |

Graphics



$T_{film}|θ=-20°$
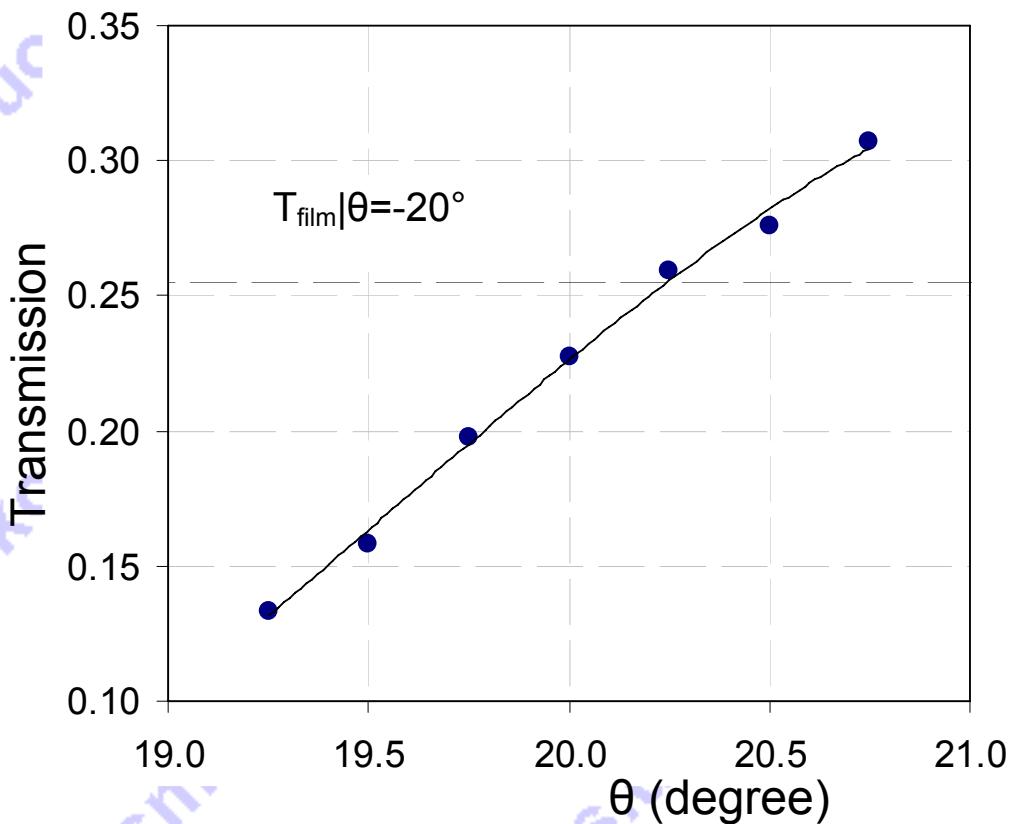
We see that: T($\theta$ = 20.25˚) = T($\theta$ = -20˚)

| $\delta$ (degree) | 0.25±0.08 |
|---|---|

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2a.

$$\lambda = d \sin\left(\theta - \frac{\delta}{2}\right) \Rightarrow \Delta\lambda = \lambda\sqrt{\left(\frac{\Delta d}{d}\right)^2 + \cot^2\left(\theta - \frac{\delta}{2}\right)\left(\Delta\theta^2 + \frac{\Delta\delta^2}{4}\right)} \approx d\,\cos(\theta)\left(\frac{0.1\pi}{180}\right)$$

where $\Delta\theta = \Delta\delta = 5' = 0.08$ degree

and $d = \dfrac{1}{600}$ mm

$$\boxed{\Delta\lambda = 2.9\,\cos(\theta)\ \text{(nm)}}$$

$$T_{film} = \frac{R_{glass}}{R_{film}} \Rightarrow \Delta T = T_{film}\sqrt{\left(\frac{\Delta R_{film}}{R_{film}}\right)^2 + \left(\frac{\Delta R_{glass}}{R_{glass}}\right)^2}$$

$$\boxed{\Delta T = \frac{R_{glass}}{R_{film}}\sqrt{\left(\frac{\Delta R_{film}}{R_{film}}\right)^2 + \left(\frac{\Delta R_{glass}}{R_{glass}}\right)^2}}$$

$$(\alpha t) = -Ln(T_{film}) \Rightarrow \Delta(\alpha t) = \frac{\Delta T}{T_{film}} = \sqrt{\left(\frac{\Delta R_{film}}{R_{film}}\right)^2 + \left(\frac{\Delta R_{glass}}{R_{glass}}\right)^2}$$

$$\boxed{\Delta(\alpha t) = \sqrt{\left(\frac{\Delta R_{film}}{R_{film}}\right)^2 + \left(\frac{\Delta R_{glass}}{R_{glass}}\right)^2}}$$

2b.

$$13 \le \theta \le 26$$

| $\Delta\lambda$ (nm) | 2.6-2.8 |
|---|---|

2c.

Table 2c. The calculated parameters using the measured parameters

| $\theta$ (degree) | $\lambda$ (nm) | $I_g/C(\lambda)$ $(M\Omega^{-1})$ | $I_s/C(\lambda)$ $(M\Omega^{-1})$ | $T_{film}$ | $\alpha t$ | $\Delta(\alpha t)$ |
|---|---|---|---|---|---|---|
| 15.0 | 428 | 0.265 | 0.00546 | 0.0206 | 3.88 | 0.02 |
| 15.5 | 442 | 0.388 | 0.00758 | 0.0195 | 3.94 | 0.02 |
| 16.0 | 456 | 0.532 | 0.0115 | 0.0216 | 3.83 | 0.01 |
| 16.25 | 463 | 0.725 | 0.0150 | 0.0208 | 3.88 | 0.01 |
| 16.5 | 470 | 0.840 | 0.0194 | 0.0231 | 3.77 | 0.01 |
| 16.75 | 477 | 1.00 | 0.0250 | 0.0250 | 3.69 | 0.01 |
| 17.0 | 484 | 1.12 | 0.0299 | 0.0266 | 3.63 | 0.01 |
| 17.25 | 491 | 1.39 | 0.0427 | 0.0308 | 3.48 | 0.02 |
| 17.5 | 498 | 1.47 | 0.0515 | 0.0351 | 3.35 | 0.02 |
| 17.75 | 505 | 1.87 | 0.0781 | 0.0418 | 3.17 | 0.01 |
| 18.0 | 512 | 2.06 | 0.096 | 0.0467 | 3.06 | 0.01 |
| 18.25 | 518 | 2.56 | 0.146 | 0.0572 | 2.86 | 0.01 |
| 18.5 | 525 | 2.74 | 0.185 | 0.0676 | 2.69 | 0.01 |
| 18.75 | 532 | 3.41 | 0.289 | 0.0847 | 2.47 | 0.01 |
| 19.0 | 539 | 3.65 | 0.376 | 0.103 | 2.27 | 0.01 |
| 19.25 | 546 | 4.26 | 0.568 | 0.134 | 2.01 | 0.01 |
| 19.5 | 553 | 4.44 | 0.704 | 0.158 | 1.84 | 0.01 |
| 19.75 | 560 | 5.13 | 1.01 | 0.197 | 1.62 | 0.01 |
| 20.0 | 567 | 5.00 | 1.14 | 0.227 | 1.48 | 0.01 |
| 20.25 | 573 | 4.98 | 1.29 | 0.259 | 1.35 | 0.01 |
| 20.5 | 580 | 4.41 | 1.22 | 0.276 | 1.29 | 0.01 |
| 20.75 | 587 | 3.66 | 1.12 | 0.307 | 1.18 | 0.02 |
| 21.0 | 594 | 2.72 | 0.890 | 0.328 | 1.12 | 0.01 |
| 21.5 | 607 | 1.67 | 0.621 | 0.373 | 0.99 | 0.01 |
| 22.0 | 621 | 1.29 | 0.541 | 0.419 | 0.87 | 0.01 |
| 22.5 | 634 | 1.20 | 0.535 | 0.444 | 0.81 | 0.01 |
| 23.0 | 648 | 1.14 | 0.518 | 0.456 | 0.79 | 0.02 |
| 23.5 | 661 | 0.99 | 0.467 | 0.472 | 0.75 | 0.01 |
| 24.0 | 675 | 0.826 | 0.388 | 0.469 | 0.76 | 0.01 |
| 24.5 | 688 | 0.649 | 0.306 | 0.471 | 0.75 | 0.01 |
| 25.0 | 701 | 0.524 | 0.242 | 0.462 | 0.77 | 0.01 |

2d.

Graphics
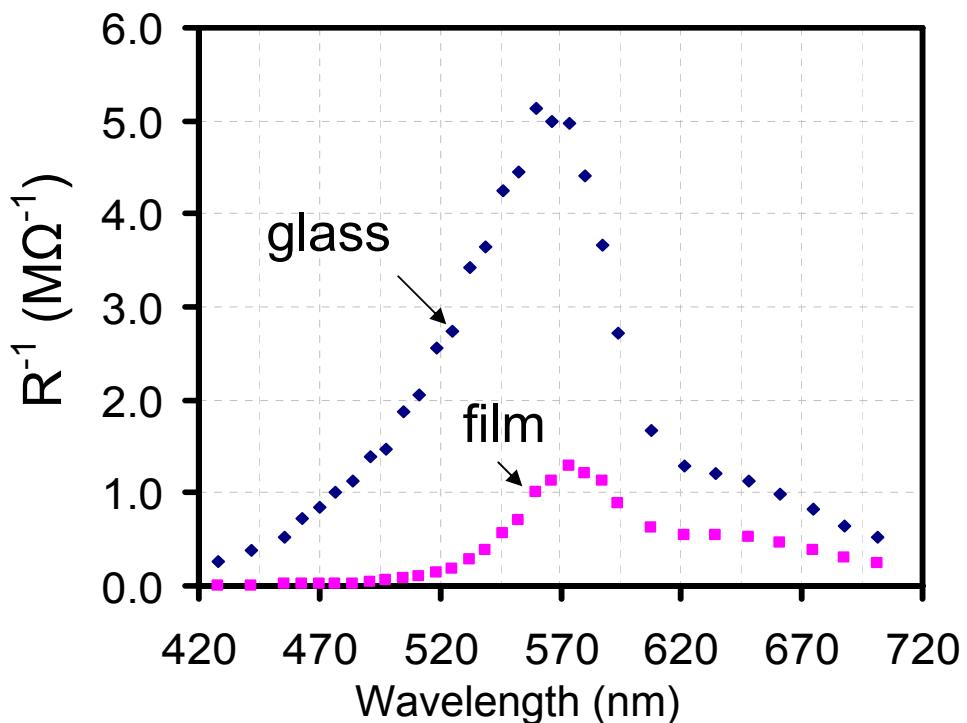


| $\lambda_{max}(I_{glass})$ | 564±5 (nm) |
|---|---|
| $\lambda_{max}(I_{film})$ | 573±5 (nm) |

2e. Graphics



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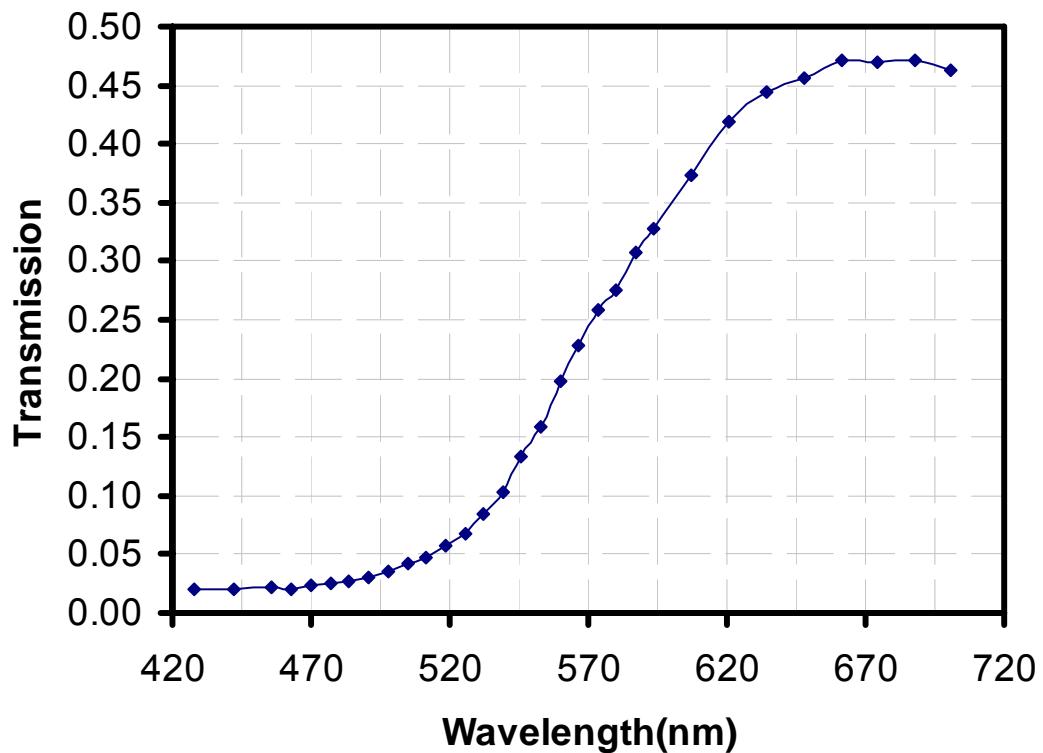
## Task 3.

3a.

$$x = h\nu = \frac{hc}{\lambda} \Rightarrow \frac{\Delta x}{x} = \frac{\Delta \lambda}{\lambda}$$

$$y = (x(\alpha t))^2 \Rightarrow \frac{\Delta y}{y} = 2\sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta(\alpha t)}{(\alpha t)}\right)^2}$$

$$\frac{\Delta x}{x} = \frac{\Delta \lambda}{\lambda}$$

$$\frac{\Delta y}{y} = 2\sqrt{\left(\frac{\Delta \lambda}{\lambda}\right)^2 + \left(\frac{\Delta(\alpha t)}{(\alpha t)}\right)^2}$$

3b.

Table 3b. The calculated parameters for each measured data point

| $\theta$ (degree) | $x$ (eV) | $y$ ( eV$^2$) | $\Delta x$ (eV) | $\Delta y$ (eV$^2$) |
|---|---|---|---|---|
| 15.00 | 2.898 | 126.6 | 0.019 | 2.0 |
| 15.50 | 2.806 | 121.9 | 0.018 | 1.9 |
| 16.00 | 2.720 | 108.8 | 0.017 | 1.5 |
| 16.25 | 2.679 | 107.8 | 0.016 | 1.4 |
| 16.50 | 2.639 | 98.9 | 0.016 | 1.3 |
| 16.75 | 2.600 | 92.0 | 0.015 | 1.2 |
| 17.00 | 2.563 | 86.3 | 0.015 | 1.2 |
| 17.25 | 2.527 | 77.4 | 0.014 | 1.1 |
| 17.50 | 2.491 | 69.7 | 0.014 | 1.0 |
| 17.75 | 2.457 | 60.9 | 0.013 | 0.8 |
| 18.00 | 2.424 | 55.1 | 0.013 | 0.8 |
| 18.25 | 2.392 | 46.8 | 0.013 | 0.6 |
| 18.50 | 2.360 | 40.4 | 0.012 | 0.6 |
| 18.75 | 2.330 | 33.1 | 0.012 | 0.5 |
| 19.00 | 2.300 | 27.3 | 0.012 | 0.4 |
| 19.25 | 2.271 | 20.91 | 0.011 | 0.36 |
| 19.50 | 2.243 | 17.07 | 0.011 | 0.27 |
| 19.75 | 2.215 | 12.92 | 0.011 | 0.22 |
| 20.00 | 2.188 | 10.51 | 0.010 | 0.19 |
| 20.25 | 2.162 | 8.53 | 0.010 | 0.17 |
| 20.50 | 2.137 | 7.56 | 0.010 | 0.14 |
| 20.75 | 2.112 | 6.23 | 0.010 | 0.18 |
| 21.00 | 2.088 | 5.43 | 0.009 | 0.11 |
| 21.50 | 2.041 | 4.06 | 0.009 | 0.09 |
| 22.00 | 1.997 | 3.02 | 0.009 | 0.06 |
| 22.50 | 1.954 | 2.52 | 0.008 | 0.08 |
| 23.00 | 1.914 | 2.26 | 0.008 | 0.09 |
| 23.50 | 1.875 | 1.98 | 0.008 | 0.07 |
| 24.00 | 1.838 | 1.94 | 0.007 | 0.06 |
| 24.50 | 1.803 | 1.84 | 0.007 | 0.05 |
| 25.00 | 1.769 | 1.86 | 0.007 | 0.04 |

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3c.

Graphics



| $x_{min} = 2.24 (\text{eV})$ | $x_{max} = 2.68 (\text{eV})$ |
|---|---|
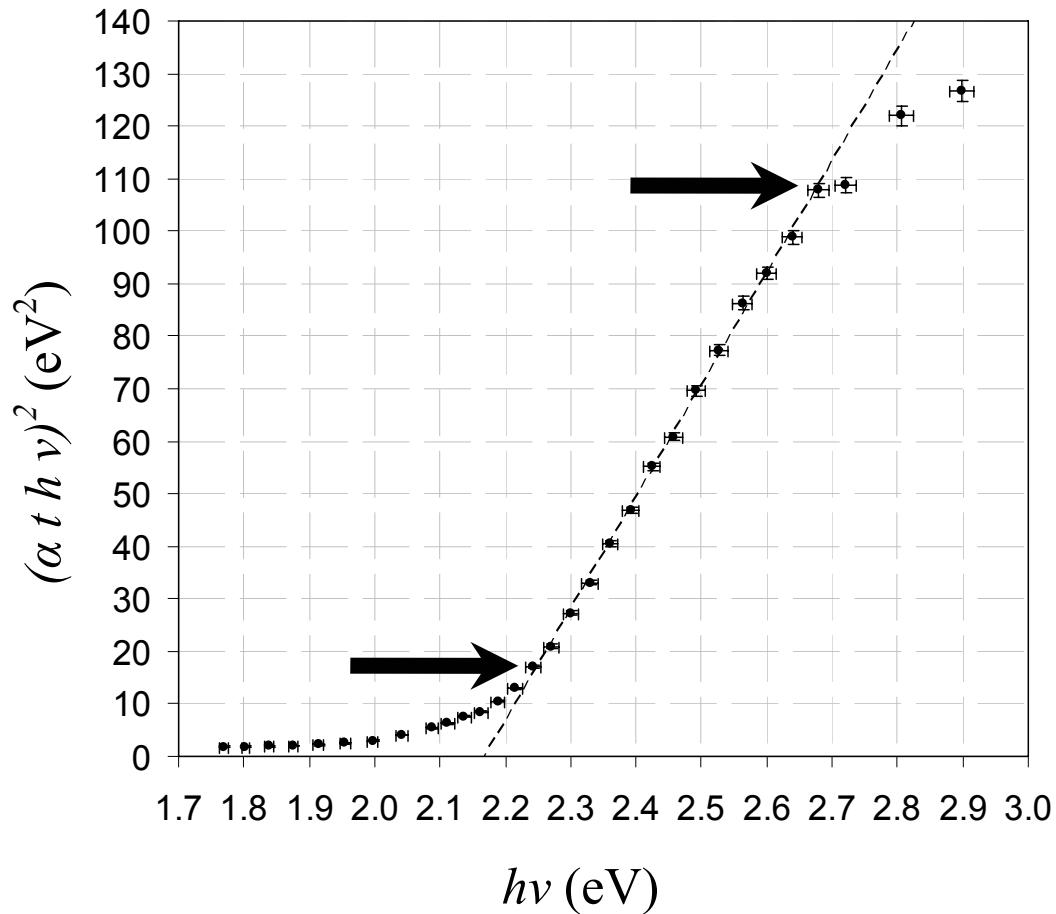
3d.

$$\alpha h\nu = A\left(h\nu - E_g\right)^{\frac{1}{2}} \Rightarrow \left(\alpha t\, h\nu\right)^2 = \left(At\right)^2\left(h\nu - E_g\right)$$

$$\Rightarrow y = \left(At\right)^2\left(x - E_g\right) \Rightarrow m = \left(At\right)^2 \Rightarrow t = \frac{\sqrt{m}}{A}$$

$$\Rightarrow \frac{\Delta t}{t} = \frac{\Delta m}{2m}$$

| |
|---|
| $t = \dfrac{\sqrt{m}}{A}$ |
| $\Delta t = \dfrac{\Delta m}{2A\sqrt{m}}$ |

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In linear range we have, m=213 (eV), $r^2$= 0.9986, $E_g$=2.17 (eV)
and we have $A = 0.071 \left(eV^{1/2}/nm\right)$ so we find t= 206 (nm)

$$\Delta m = \sqrt{\frac{(\delta y)^2 + \frac{m^2}{R^2}(\delta x)^2}{\sum_i x_i^2 - N\overline{x}^2}} \approx \sqrt{\frac{(\delta y)^2 + (m\,\delta x)^2}{\sum_i x_i^2 - N\overline{x}^2}} = \sqrt{\frac{(\delta xy)^2}{\sum_i x_i^2 - N\overline{x}^2}}, \; (\delta xy)^2 = (\delta y)^2 + (m\,\delta x)^2$$

where δx & δy are the mean of error range of x & y

$$\delta x \approx \sqrt{\frac{\sum_i \delta x_i^2}{N}} \; \& \; \delta y = \sqrt{\frac{\sum_i \delta y_i^2}{N}} \; \text{So} \qquad \delta x \approx 0.014 \; (eV), \; \delta y \approx 0.9 (eV)^2$$

→ $\Delta m \approx 10$ (eV) → $\Delta t = t \times \Delta m / (2\,m) \approx 5$ (nm)

$$\Delta E_g = \frac{1}{m} \sqrt{\left(\left(\frac{m^2\,\delta x^2 + \delta y^2}{N}\right) + \left(\frac{\overline{y}}{m}\right)^2 \Delta m^2\right)} = \frac{1}{m} \sqrt{\left(\left(\frac{\delta xy^2}{N}\right) + \left(\frac{\overline{y}}{m}\right)^2 \Delta m^2\right)}$$

$\Delta E_g \approx 0.02 (eV)$

Table 3d. The calculated values of $E_g$ and $t$ using Fig. 3

| $E_g$ (eV) | $\Delta E_g$ (eV) | $t$ (nm) | $\Delta t$ (nm) |
|---|---|---|---|
| 2.17 | 0.02 | 206 | 5 |

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