1^{st} Question "Pink"

1.1
Period = 3.0 days = 2.6×10^5 s .
Period = $\dfrac{2\pi}{\omega}$ $\Rightarrow$ $\omega = 2.5\times10^{-5}$ rad s^{-1}.

1.2
Calling the minima in the diagram 1, $I_1/I_0 = \alpha = 0.90$ and $I_1/I_0 = \beta = 0.63$, we have:

$$\frac{I_0}{I_1} = 1+\left(\frac{R_2}{R_1}\right)^2\left(\frac{T_2}{T_1}\right)^4 = \frac{1}{\alpha}$$

$$\frac{I_2}{I_1} = 1-\left(\frac{R_2}{R_1}\right)^2\left(1-\left(\frac{T_2}{T_1}\right)^4\right) = \frac{\beta}{\alpha}$$

From above, one finds:

$$\frac{R_1}{R_2} = \sqrt{\frac{\alpha}{1-\beta}} \Rightarrow \frac{R_1}{R_2} = 1.6 \qquad \text{and} \qquad \frac{T_1}{T_2} = \sqrt[4]{\frac{1-\beta}{1-\alpha}} \Rightarrow \frac{T_1}{T_2} = 1.4$$

2.1)
Doppler-Shift formula:
$\dfrac{\Delta\lambda}{\lambda_0} \cong \dfrac{v}{c}$ (or equivalent relation)

Maximum and minimum wavelengths: $\lambda_{1,\max} = 5897.7$ Å , $\lambda_{1,\min} = 5894.1$ Å
$\lambda_{2,\max} = 5899.0$ Å , $\lambda_{2,\min} = 5892.8$ Å

Difference between maximum and minimum wavelengths:
$\Delta\lambda_1 = 3.6$ Å , $\Delta\lambda_2 = 6.2$ Å

Using the Doppler relation and noting that the shift is due to twice the orbital speed:

$$v_1 = c\frac{\Delta\lambda_1}{2\lambda_0} = 9.2\times10^4 \text{ m/s}$$

$$v_2 = c\frac{\Delta\lambda_2}{2\lambda_0} = 1.6\times10^5 \text{ m/s}$$

2.2) As the center of mass is not moving with respect to us:

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = 1.7$$

2.3)

Writing $r_i = \frac{v_i}{\omega}$ for $i = 1,2$, we have

$r_1 = 3.8 \times 10^9$ m, $r_2 = 6.5 \times 10^9$ m

2.4)

$r = r_1 + r_2 = 1.0 \times 10^{10}$ m

3.1)

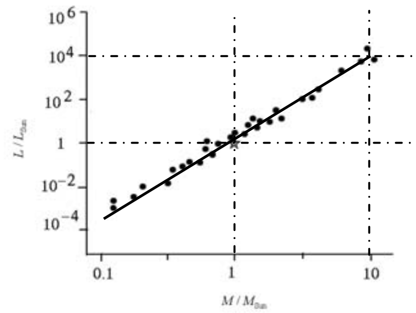
The gravitational force is equal to mass times the centrifugal acceleration

$$G\frac{m_1 m_2}{r^2} = m_1 \frac{v_1^{\ 2}}{r_1} = m_2 \frac{v_2^{\ 2}}{r_2}$$

Therefore,

$$\begin{cases} m_1 = \dfrac{r^2 v_2^{\ 2}}{G r_2} \\ m_2 = \dfrac{r^2 v_1^{\ 2}}{G r_1} \end{cases} \Rightarrow \begin{cases} m_1 = 6 \times 10^{30} \text{kg} \\ m_2 = 3 \times 10^{30} \text{kg} \end{cases}$$

4.1) As it is clear from the diagram, with one significant digit, $\alpha = 4$.



4.2)

As we have found in the previous section: $L_i = L_{Sun}\left(\frac{M_i}{M_{Sun}}\right)^4$

So,

$$L_1 = 5\times10^{28}\,\text{Watt}$$
$$L_2 = 6\times10^{27}\,\text{Watt}$$

4.3) The total power of the system is distributed on a sphere with radius d to produce I_0, that is:

$$I_0 = \frac{L_1 + L_2}{4\pi d^2} \qquad \Rightarrow d = \sqrt{\frac{L_1 + L_2}{4\pi I_0}} = 1\times10^{18}\,\text{m}$$
$$= 100\ \text{ly.}$$

4.4) $$\theta \cong \tan\theta = \frac{r}{d} = 1\times10^{-8}\,\text{rad.}$$

4.5)
A typical optical wavelength is λ_0. Using uncertainty relation:

$$D = \frac{d\,\lambda_0}{r_{tot}} = 50\ \text{m.}$$