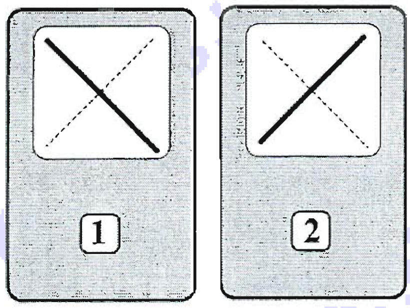
Experimental problem. To see invisible!

Part 1. Watch!

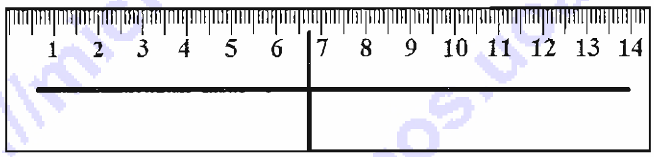
Section 1.1. Polarizers

For determination of the transmission plane of the polarizer, one can use a glaring effect from any shining surface. It is known that the reflected light is polarized in the plane of the reflecting surface. The corresponding transmission planes are shown in the figure on the right.



Section 1.2. Rulers

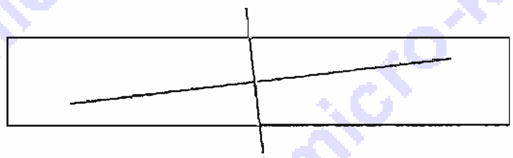
1.2.1. In case of the incident light being polarized along the optical axis or perpendicular to it, there is only one kind of waves generated in the medium. This means that no change in light polarization is to occur. Thus, it is possible to determine either the direction of the optical axis or the direction which is perpendicular to it. Those possible alternatives are shown in the figure above (either along the ruler, or perpendicular to it).



1.2.2. One can see at what parts of the rulers similar colors are observed, mainly with blue hue.
The distance between those bands for the ruler No.1 is ~12 cm, while for the two rules stacked together it is ~ 8 cm.

Section 1.3. Strip

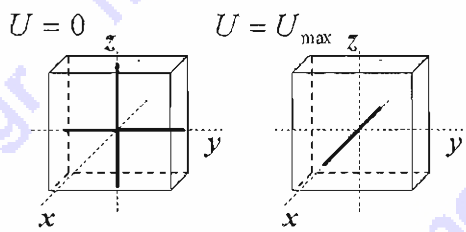
1.3.1 Possible directions of the optical axis of the strip can be determined in a similar way. As shown in the figure on the right, those directions make a small angle $\approx 10°$ with the sides of the strip.
1.3.2 The coordinates of the dark bands are approximately found as follows $x_L = 3{,}5cm$, $x_R = 7{,}5cm$.



Section 1.4. Liquid crystal cell

1.4.1 In case of zero voltage, the directions of the optical axis can be determined in the same way: It is either horizontal or vertical.
At the maximum voltage applied the optical axis orients along the electric field, which means it turns perpendicular to the cell plane.
1.4.2 The voltage at which suach a sharp transition in orientation of molecules of the liquid crystal occurs is approximately equal to

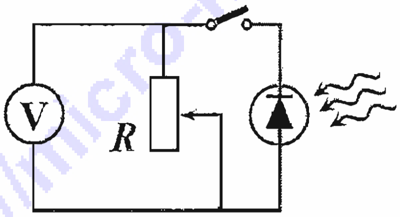
$$U_{cr} = 2\text{ V}.$$



Part 2. Measure!

Section 2.1. Investigating a photodiode

2.1.1. In the figure below a position for a circuit switch is shown. During measurements of the resistance, the circuit switch should be unshorted.

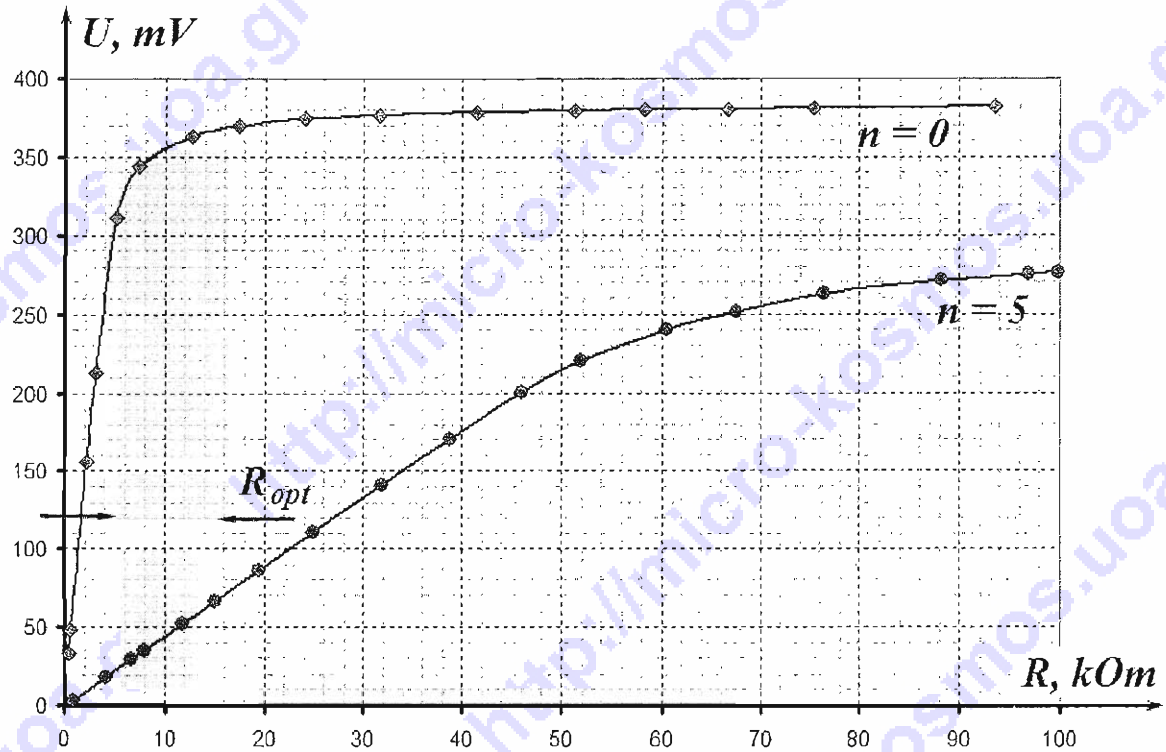


2.1.2 In table 1 the results are presented of the measurements of the voltage U as a function of the resistance. Those data are plotted in the corresponding graph.

Table 1.

$n = 0$		$n = 5$	
$R, 10^3 Ohm$	$U, 10^{-3} V$	$R, 10^3 Ohm$	$U, 10^{-3} V$
0,4	33	0,9	3
0,6	48	4,1	18
2,2	156	6,6	29
3,1	213	8,0	35
5,1	311	11,8	52
7,4	344	15,0	66
12,7	363	19,4	86
17,4	370	24,9	111
24,0	374	31,8	141
31,5	376	38,8	170
41,5	378	46,0	200
51,4	379	51,9	220
58,3	380	60,4	240
66,6	380	67,5	252
75,4	381	76,4	263
93,5	382	88,2	271
		96,9	275
		99,8	276

Note that the optimal resistance should be within the range 5-15 $kOhm$, which corresponds to the largest variation in the voltage.



2.1.3 In table 2 the results are shown of the measurements for the voltage as a function of the number of light filters at different values of resistance.

Table 2.

$R =$	5,1 kOhm		29,9 kOhm		20,4 kOhm		10,1 kOhm	
n	U, mV	$\ln U$	U, mV	$\ln U$	U, mV	$\ln U$	U, mV	$\ln U$
0	351	5,861	391	5,969	388	5,961	377	5,932
1	290	5,670	370	5,914	364	5,897	341	5,832
2	168	5,124	346	5,846	336	5,817	294	5,684
3	92	4,522	317	5,759	309	5,733	179	5,187
4	56	4,025	288	5,663	234	5,455	105	4,654
5	35	3,555	212	5,357	148	4,997	66	4,190

Intensity of the light I_n that has passed through the filter decreases as a geometric progression when increasing the number of filters n:

$$I_n = I_0 \gamma^n. \quad (1)$$

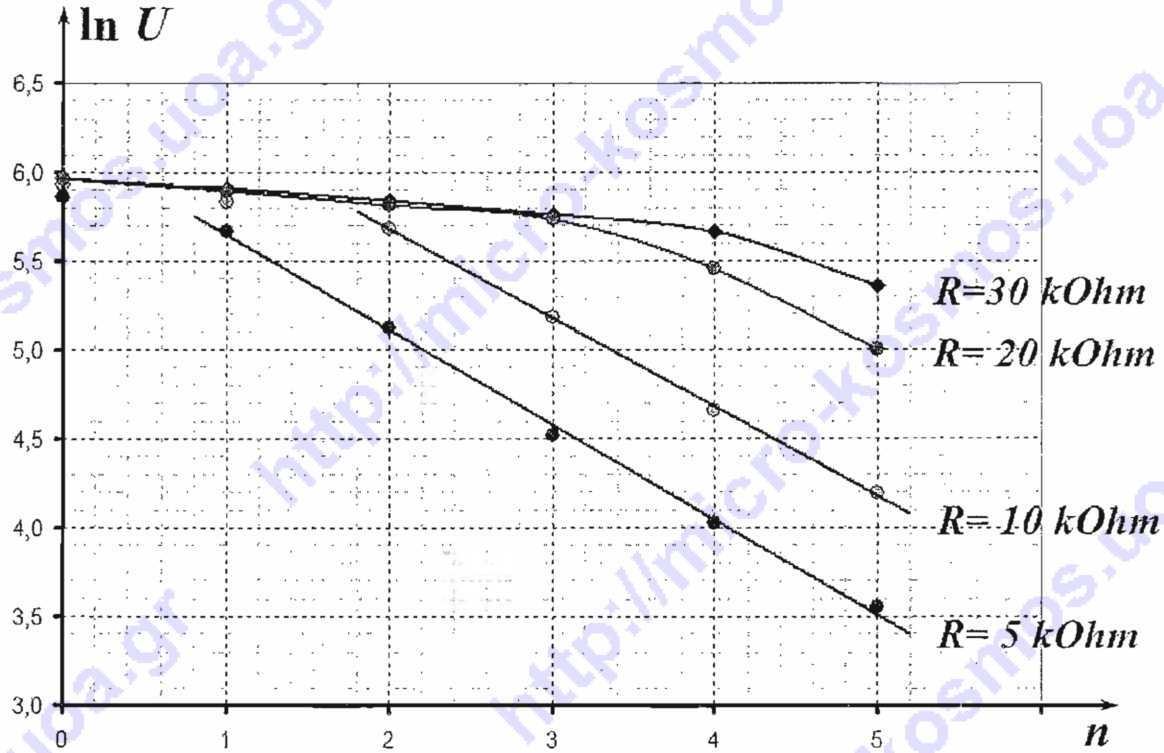
In case if the measured voltage is proportional to the intensity of the incident light, it obeys a similar law:

$$U_n = U_0 \gamma^n. \quad (2)$$

To verify equation (2), one needs to use a semi-logariphmic scale. In other words, it is necessary to plot $\ln U$ as a function of n:

$$\ln U_n = \ln U_0 + n \ln \gamma. \quad (3)$$

That plot is shown in the following figure.



According to the graph above, by decreasing the resistance the dependence turns a linear function. It means, firstly, in order to decrease deviation in transmission coefficient γ, we can make additional measurements at lower resistances (here values for R= 10 Ohm are shown); secondly, further measurements should be made at the lowest resistance among given values, i.e. at $R = 5\ kOhm$.

According to equation (3) , the slope is $\alpha = \ln\gamma$. Using the Method of Least Squares, we can obtain its value $\alpha = -0.53 \pm 0.03$. Thus, the coefficient of transmission turns to be equal to $\gamma = \exp\alpha = 0.59$ with an error, which can be calculated by applying the following formula $\Delta\gamma = \exp(\alpha)\Delta\alpha = 0.02$. Finally we obtain

$$\gamma = 0.59 \pm 0.02.$$

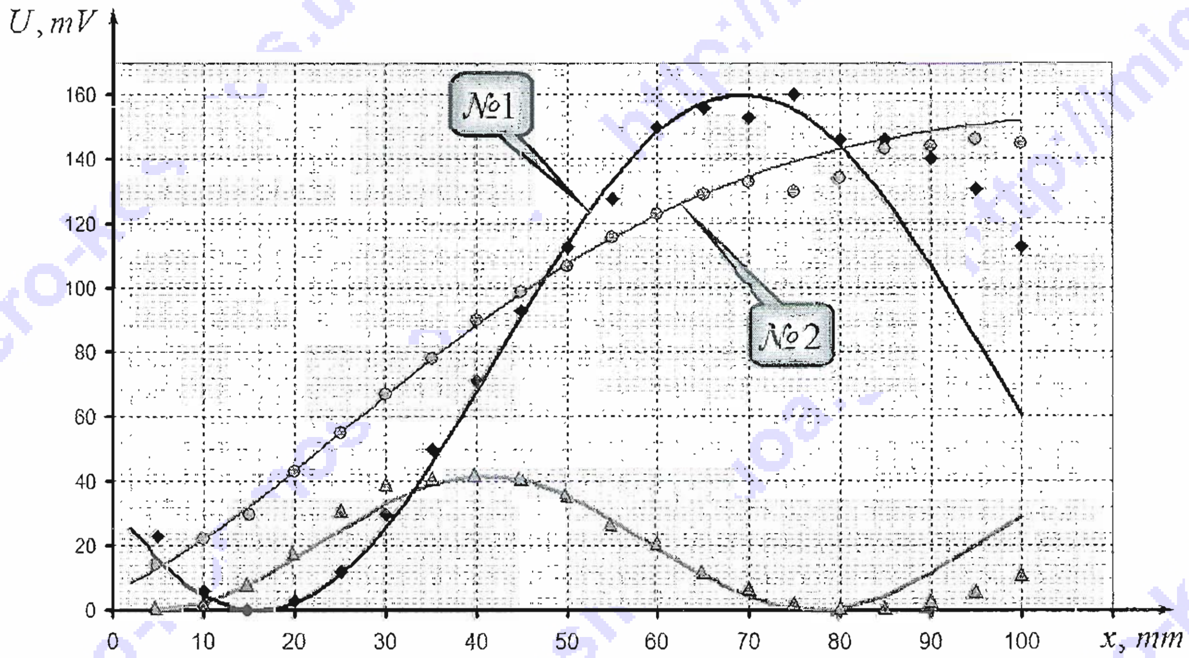
Note that values for $R = 10\ kOhm$ *produce the following result:* $\gamma = 0.59 \pm 0.02$.

Часть 2.2 Light transition through a plastic ruler

2.2.1 Results of measurements of the light intensity as a function of coordinates of transmission points through ruler #1, #2 and both rulers, are shown in table 3 and in the graph below.

Table 3.

№1			№2			Both rulers		
X,mm	*U,mV*	$\Delta\varphi$	*X,mm*	*U,mV*	$\Delta\varphi$	*X,mm*	*U,mV*	U_{calc}
5	23	0,778	5	14	0,601	5	1	0,0
10	6	0,390	10	22	0,760	10	2	2,1
15	0	0,000	15	30	0,896	15	8	7,7
20	3	0,275	20	43	1,090	20	18	15,7
25	12	0,555	25	55	1,253	25	31	24,7
30	30	0,896	30	67	1,408	30	39	33,0
35	50	1,186	35	78	1,546	35	41	39,2
40	71	1,458	40	90	1,696	40	42	41,9
45	93	1,734	45	99	1,811	45	41	40,8
50	113	1,996	50	107	1,915	50	36	36,0
55	128	2,214	55	116	2,038	55	27	28,5
60	150	2,636	60	123	2,138	60	21	19,6
65	156	2,824	65	129	2,230	65	12	10,9
70	153	2,720	70	133	2,295	70	7	4,1
75	160	3,142	75	130	2,246	75	2	0,4
80	146	2,541	80	134	2,312	80	1	0,5
85	146	2,541	85	143	2,478	85	1	4,4
90	140	2,419	90	144	2,498	90	3	11,4
95	131	2,262	95	146	2,541	95	6	20,1
100	113	1,996	100	145	2,519	100	11	29,0



2.2.2 To calculate a phase shift, we use equation (1), mentioned in the problems formulation, which can be represented as

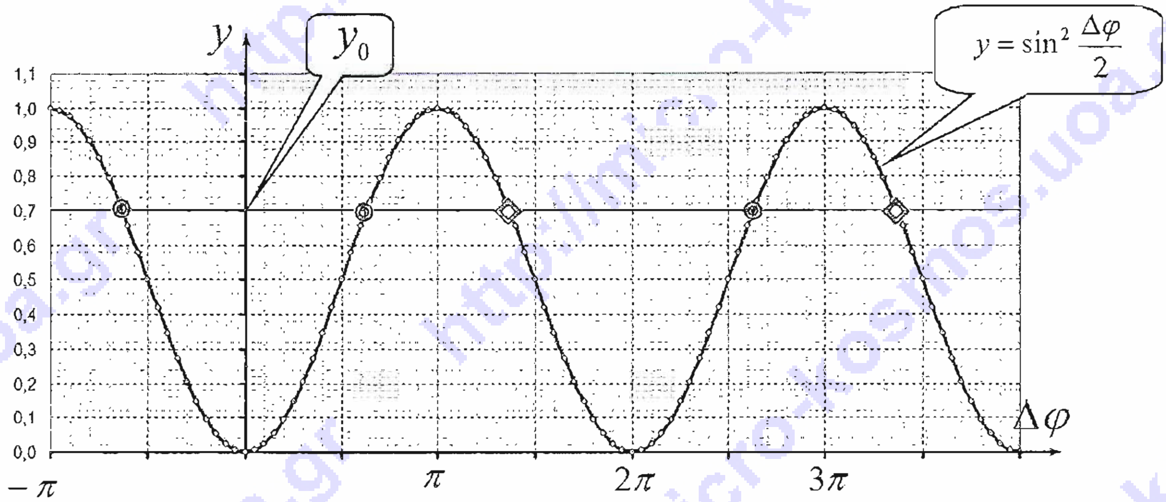
$$U = U_{max}\sin^2\frac{\Delta\varphi}{2}, \qquad (1)$$

where U_{max} is the largest value of voltage. But we have to be sure that this value actually corresponds to the maximum of function (1), not just another boundary point. According to measurements, (see the graph) for each ruler the most suitable value for U_{max} is $U_{max} = 160\ mV$.

The following equation

$$y_0 = \sin^2\frac{\Delta\varphi}{2} \qquad (1)$$

has multiple roots and it is not easy to find actual values of the phase shift, even if it's possible to calculate certain value for U_{max}, roots of the equation mentioned above are shown in the figure below



Formally we can represent the roots in different forms, for example,

$$\Delta\varphi = \pm 2(\arcsin\sqrt{y_0} + k\pi),$$
$$\Delta\varphi = \pm 2(\pi - \arcsin\sqrt{y_0} + k\pi), \qquad (2)$$
$$k = 0,1,2 \ldots.$$

Choosing a correct root should depend on a function obtained experimentally.

Values for the phase shifts calculated by the equation

$$\Delta\varphi = 2\arcsin\sqrt{\frac{U}{U_{max}}}. \qquad (3)$$

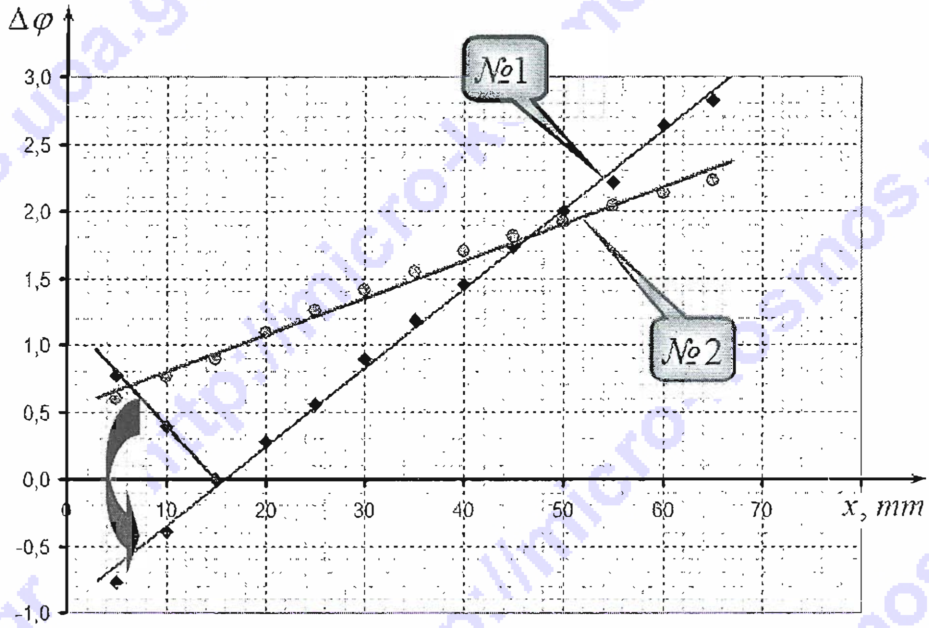
are shown in table 3

It is clear that the function $\Delta\varphi(x)$ has to be monotonous, that is why the signs of roots for the first two points must be changed, which is a mathematically correct operation (just reflecting the graph). Note that the phase shifts are calculated with uncertainty of $\pm 2\pi k$.

Obtained functions are close to linear, using MLS we get

$$\Delta\varphi_1 = 0.059x - 0.94,$$
$$\Delta\varphi_2 = 0.028x + 0.52.$$

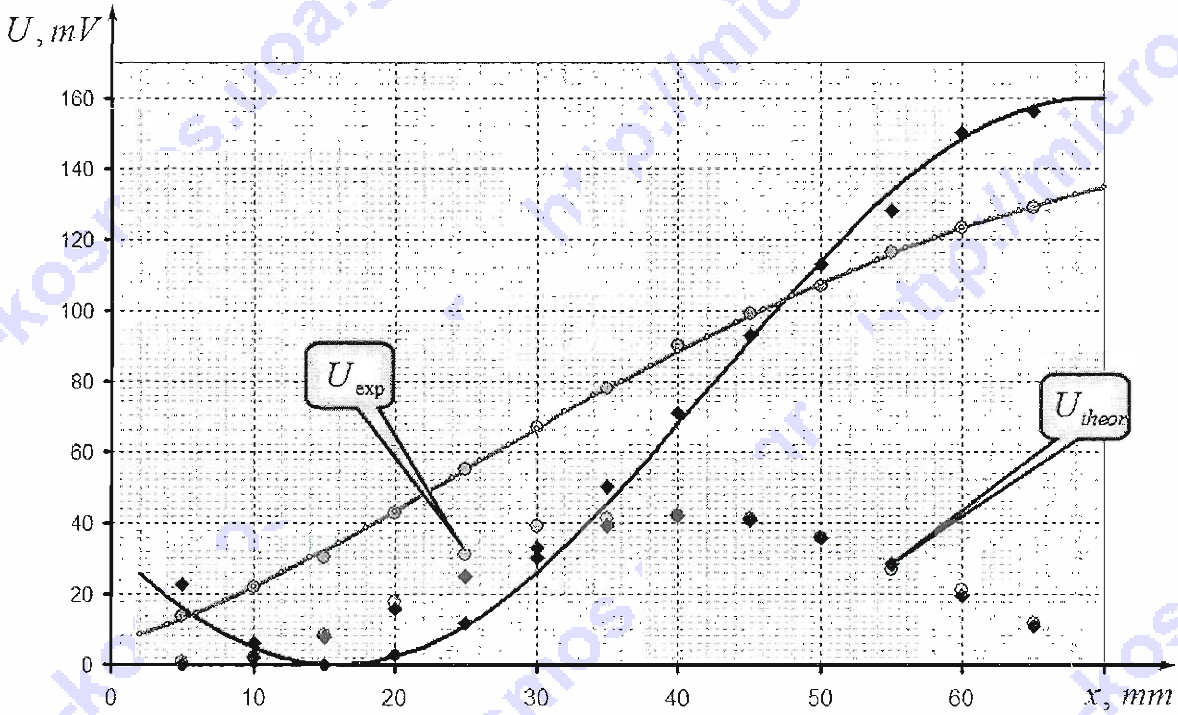
Graphs of those functions are shown below.



2.2.4 If two rulers are stacked together, then phase shifts simply add, and, theoretically, the intensity as function of phase shifts can be written as

$$U = U_{max} \sin^2 \frac{\Delta\varphi_1 + \Delta\varphi_2}{2}. \quad (5)$$

Here U_{max} is the largest value of voltage at the light transition through both rulers and can be obtained from experimental data.

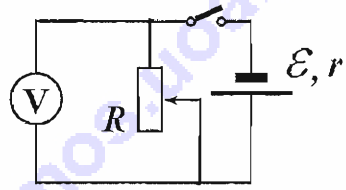


Results of the calculations are shown in table 3 and in the graph above. Consistency of theoretical calculations and experimental data can clearly be seen.

Part 2.3 Liquid crystal cell

2.3.1 Investigating power supply

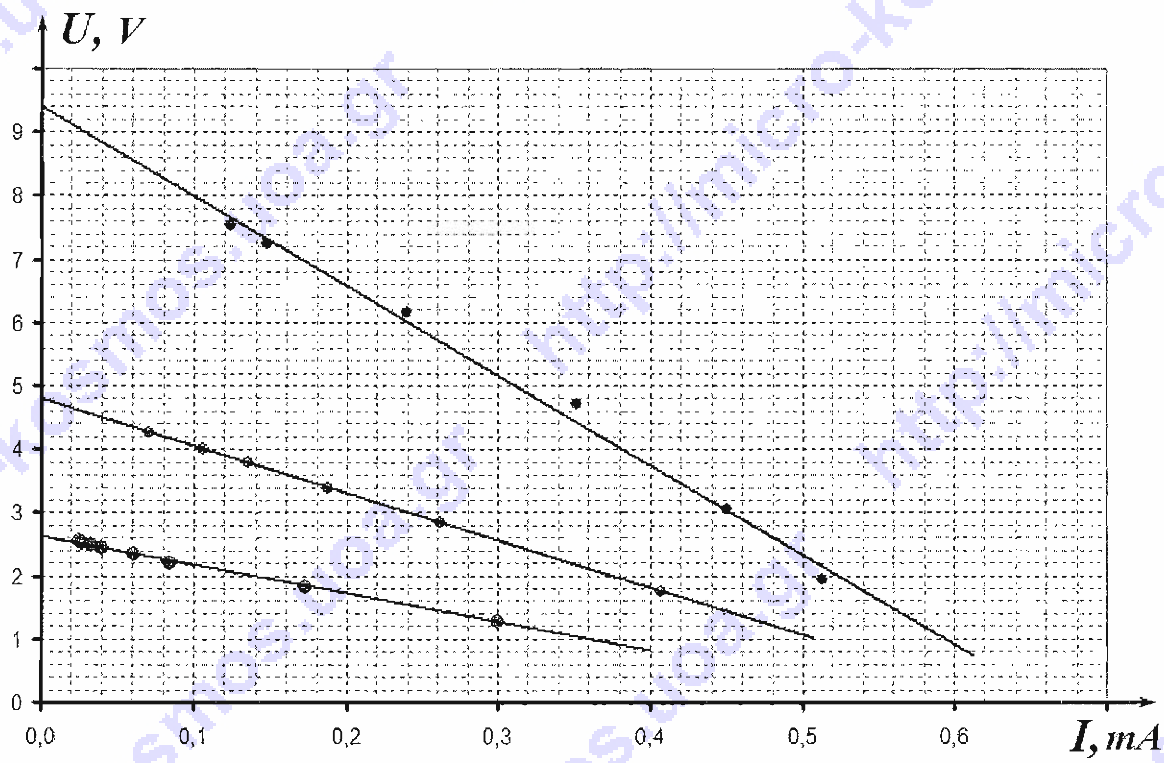
2.3.1.1 The circuit used in this part is shown in figure on the right. When the switch is shorted we can measure the voltage, otherwise, we can measure the resistance.



2.3.1.2 Results of measurements of the voltage U and the current I, and the resistance, calculated as $R=U/I$, are shown in table 4. Obtained functions are shown in the graph, where U_0 is value measured by directly connecting multimeter to the power supply.

Table 4.

$U_0=2,67$ V			$U_0=4,9$ V					
R, kOm	U, V	I, mA	R, kOm	U, V	I, mA	R, kOm	U, V	I, mA
4,3	1,29	0,300	4,3	1,75	0,407	60,8	7,53	0,124
10,6	1,83	0,173	10,9	2,85	0,261	49,1	7,24	0,147
26,2	2,20	0,084	18,2	3,40	0,187	25,7	6,15	0,239
38,5	2,34	0,061	28,1	3,80	0,135	13,4	4,72	0,352
60,1	2,43	0,040	37,8	4,01	0,106	6,8	3,06	0,450
74,7	2,48	0,033	60,6	4,26	0,070	3,8	1,95	0,513
90,7	2,52	0,028	4,3	1,75	0,407	60,8	7,53	0,124
100,0	2,55	0,026	10,9	2,85	0,261	49,1	7,24	0,147



Obtained functions are linear, it means that the internal resistance r does not depend on the current (consequently, on the external resistance), but depends on the output voltage. In this case the current is determined by Ohm's law for the circuit,

$$I = \frac{\varepsilon}{R+r}, \quad (6)$$

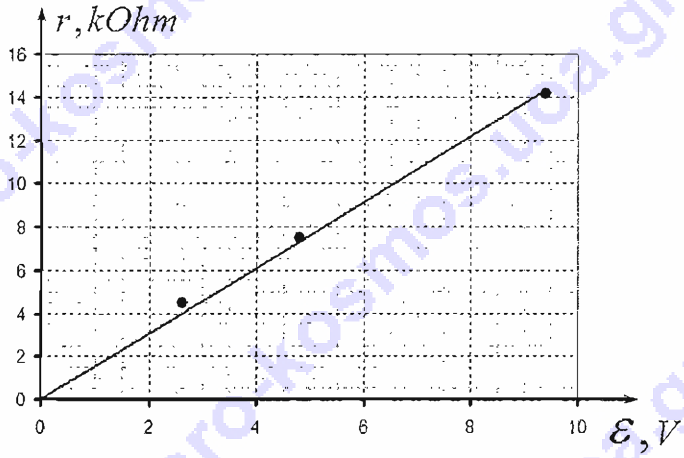
From which we get the formula for description of experimental data

$$U = IR = \varepsilon - Ir. \quad (7)$$

Parameters ε and r can be easily obtained from the graphs, as a limit value at $I \to 0$ and as a slope of the graph, respectively.

The corresponding values measured experimentally are shown in the table below together with the corresponding graph.

ε, V	$r, kOhm$
2,62	4,5
4,8	7,48
9,4	14,13



Thus, the internal resistance is proportional to EMF:

$$r = C\varepsilon, \qquad (8)$$

where the coefficient of proportionality is $C = 1.5\ \frac{kOhm}{V}$.

2.3.2 Light transmission through LCC

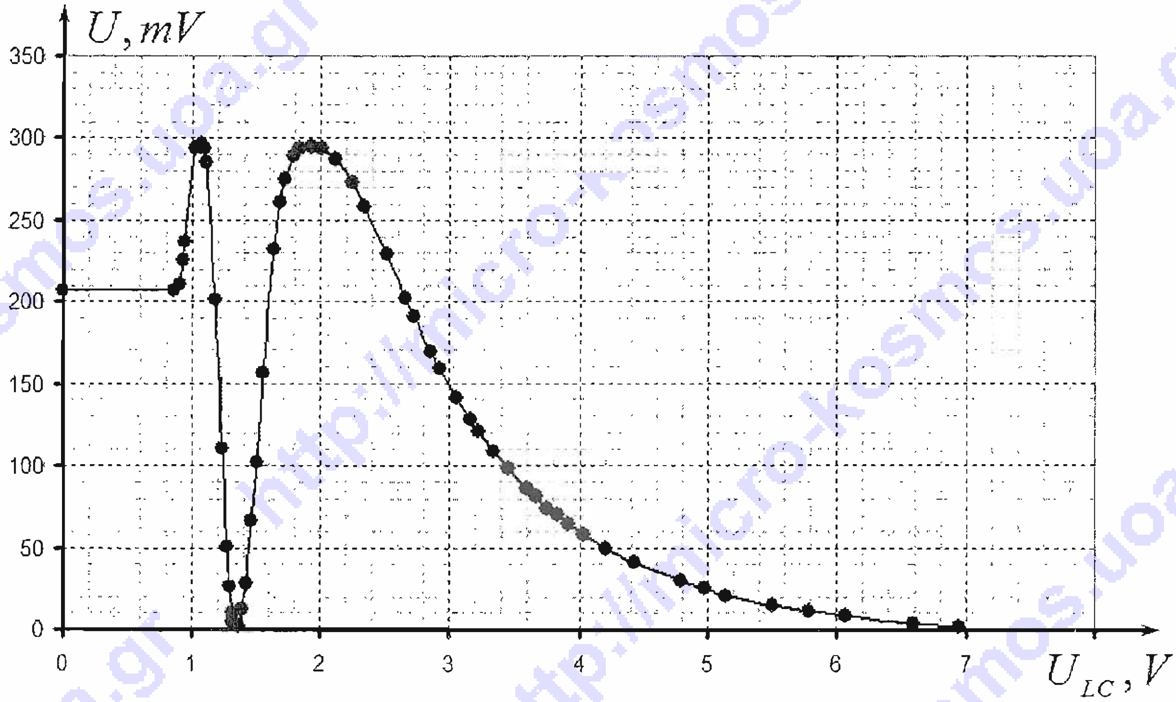
2.3.2.1 Results of the intensity measurements as a functions of voltage U_{LC} are shown in table 5[1]. Graph of the obtained function is drawn in the figure below.

Table 5.

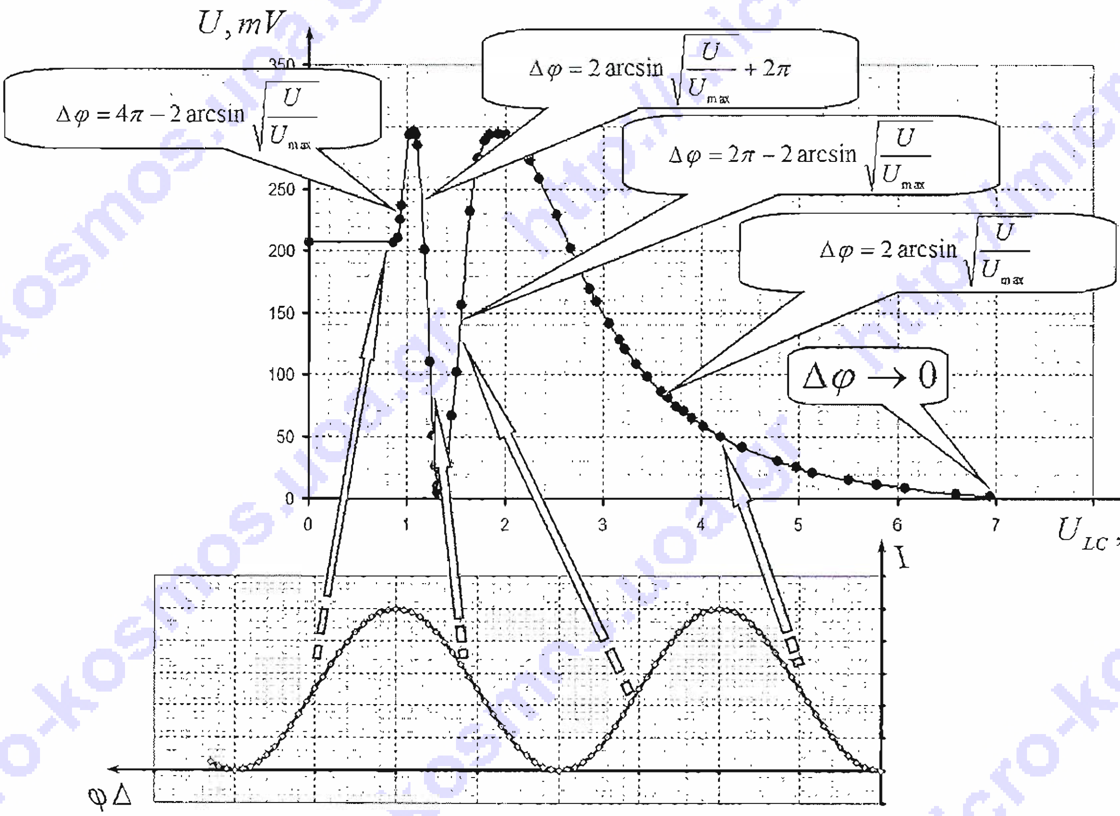
U_{LC}, V	U, mV	$(\Delta\varphi)'$	$\Delta\varphi$	$\ln U_{LC}$	$\ln \Delta\varphi$
0	207	1,961	10,606	-	2,361
0,86	207	1,961	10,606	-0,151	2,361
0,91	211	1,990	10,577	-0,094	2,359
0,93	226	2,102	10,464	-0,073	2,348
0,94	237	2,190	10,377	-0,062	2,340
1,02	294	2,858	9,709	0,020	2,273
1,07	297	2,941	9,625	0,068	2,264
1,09	294	2,858	9,709	0,086	2,273
1,11	285	2,691	8,974	0,104	2,194
1,18	201	1,918	8,201	0,166	2,104
1,23	110	1,301	7,584	0,207	2,026
1,27	51	0,850	7,133	0,239	1,965
1,29	26	0,598	6,881	0,255	1,929
1,31	10	0,367	6,650	0,270	1,895
1,32	5	0,259	6,542	0,278	1,878
1,36	2	0,163	6,447	0,307	1,864
1,39	12	0,403	5,880	0,329	1,772
1,42	28	0,621	5,662	0,351	1,734
1,46	66	0,976	5,307	0,378	1,669
1,5	102	1,245	5,038	0,405	1,617
1,55	156	1,611	4,672	0,438	1,542
1,63	232	2,149	4,134	0,489	1,419
1,68	261	2,404	3,879	0,519	1,356
1,71	275	2,556	3,727	0,536	1,316
1,78	289	2,756	3,527	0,577	1,260
1,83	294	2,858	3,425	0,604	1,231

[1] We do not expect that participants can take the same number of measurements, 15-20 points are enough. It is principally important to find the dip in the graph.

1,93	295	2,883	3,401	0,658	1,224
2,01	294	2,858	2,858	0,698	1,050
2,11	287	2,722	2,722	0,747	1,001
2,24	273	2,532	2,532	0,806	0,929
2,34	258	2,375	2,375	0,850	0,865
2,51	229	2,125	2,125	0,920	0,754
2,65	202	1,925	1,925	0,975	0,655
2,72	191	1,848	1,848	1,001	0,614
2,85	169	1,698	1,698	1,047	0,529
2,92	159	1,631	1,631	1,072	0,489
3,05	141	1,511	1,511	1,115	0,413
3,16	128	1,424	1,424	1,151	0,353
3,22	121	1,376	1,376	1,169	0,319
3,34	109	1,294	1,294	1,206	0,258
3,45	98	1,217	1,217	1,238	0,196
3,59	86	1,130	1,130	1,278	0,122
3,66	81	1,093	1,093	1,297	0,089
3,75	74	1,039	1,039	1,322	0,039
3,83	70	1,008	1,008	1,343	0,008
3,91	65	0,968	0,968	1,364	-0,032
4,03	58	0,911	0,911	1,394	-0,094
4,21	50	0,841	0,841	1,437	-0,173
4,43	41	0,757	0,757	1,488	-0,278
4,79	30	0,644	0,644	1,567	-0,441
4,98	25	0,586	0,586	1,605	-0,535
5,15	21	0,536	0,536	1,639	-0,625
5,51	15	0,451	0,451	1,707	-0,796
5,79	11	0,385	0,385	1,756	-0,954
6,07	8	0,328	0,328	1,803	-1,115
6,59	4	0,231	0,231	1,886	-1,463
6,94	2	0,163	0,163	1,937	-1,811

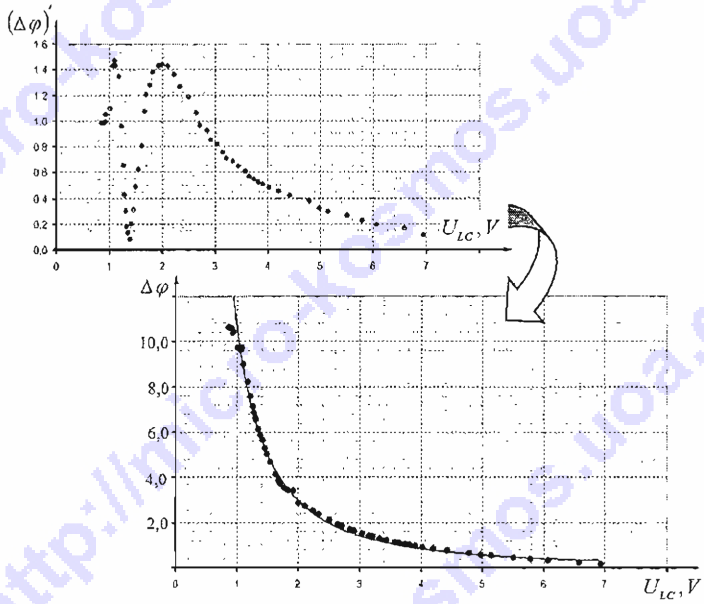


It is important to choose correct roots of equation (2) in order to adequately calculate phase shifts. In this case it is rather obviousl because at large values of U_{LC} the voltage difference tends to zero, $\Delta\varphi \to 0$.



Other solutions and corresponding equations are shown in the figure below.

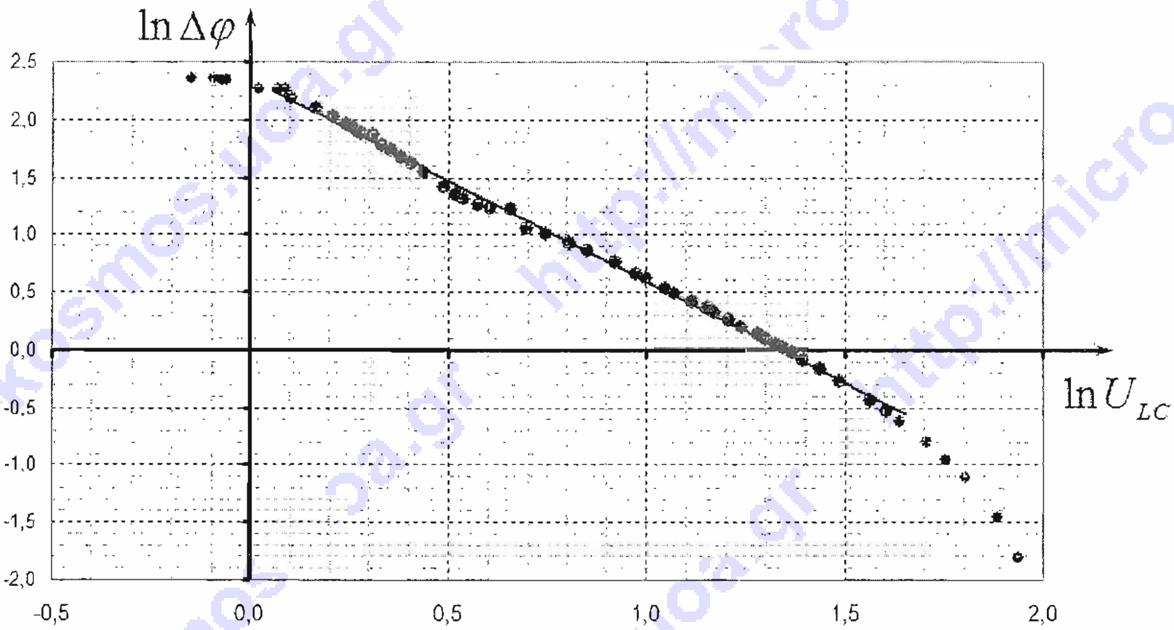
Results of calculation of $(\Delta\varphi)' = \arcsin\sqrt{\frac{U}{U_{max}}}$ and correct values of phase shift $\Delta\varphi$ are shown in the figure below.



This figure is drawn for the sake of understanding. (It's not required to draw it for participants).

The value of the phase shift at zero voltage is $\Delta\varphi_0 \approx 10.6$.

In order to check applicability of the power function $\Delta\varphi = CU^{\beta}$ it is recommended to redraw the last graph logarithmically, as shown in the figure below.



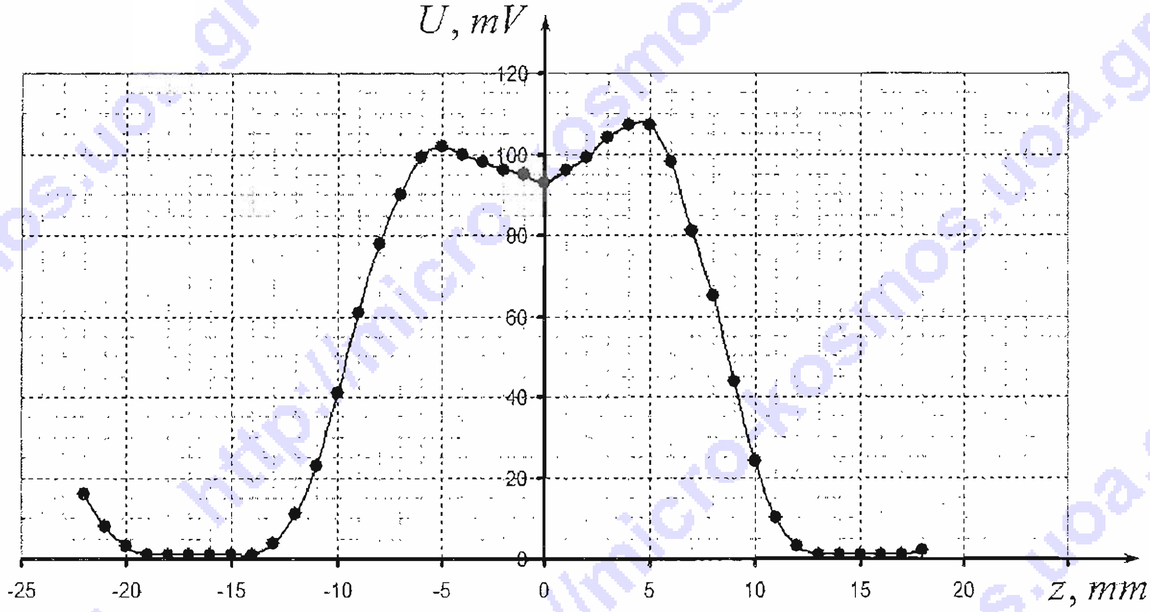
It can be seen from the graph that in the range of 1 V to 5 V the function is almost linear, which justifies the applicability of the power law. The power in that equation is equal to the slope of the graph, its numerical value is $\beta \approx 1.75$.

Section 2.4 Light transmission through a curved strip

2.4.1 Results of the measurements of the light intensity as a function of coordinate z of the point of light penetration into the strip are presented in table 6 and plotted below.

Table 6.

x, mm	U, mV	$\Delta\varphi'$	$\Delta\varphi$	z, mm
40	16	0,794	5,489	-22
41	8	0,554	5,729	-21
42	3	0,336	5,947	-20
43	1	0,194	6,090	-19
44	1	0,194	6,090	-18
45	1	0,194	6,090	-17
46	1	0,194	6,090	-16
47	1	0,194	6,090	-15
48	1	0,194	6,090	-14
49	4	0,389	5,894	-13
50	11	0,653	5,630	-12
51	23	0,964	5,319	-11
52	41	1,335	4,948	-10
53	61	1,711	4,572	-9
54	78	2,046	4,237	-8
55	90	2,322	3,962	-7
56	99	2,588	3,696	-6
57	102	2,706	2,706	-5
58	100	2,624	2,624	-4
59	98	2,553	2,553	-3
60	96	2,489	2,489	-2
61	95	2,459	2,459	-1
62	93	2,401	2,401	0
63	96	2,489	2,489	1
64	99	2,588	2,588	2
65	104	2,805	2,805	3
66	107	3,142	3,142	4
67	107	3,142	3,142	5
68	98	2,553	3,730	6
69	81	2,111	4,173	7
70	65	1,787	4,496	8
71	44	1,392	4,891	9
72	24	0,987	5,296	10
73	10	0,621	5,662	11
74	3	0,336	5,947	12
75	1	0,194	6,090	13
76	1	0,194	6,090	14
77	1	0,194	6,090	15
78	1	0,194	6,090	16
79	1	0,194	6,090	17
80	2	0,274	6,009	18



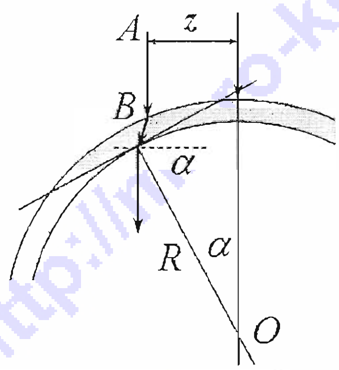
The shape of the curve indicates that $\Delta\varphi_0$ lies at the ascending part of the relation between the intensity and the phase shift, which can be calculated as

$$\Delta\varphi_0 = 10\pi + 2\arcsin\sqrt{\frac{U_0}{U_{max}}} \approx 33.9.$$

As the strip is curved, the optical length of the light depends on the coordinate where the light falls onto the strip.

Close to the center of the strip, its shape can be assumed as an arc of the circle. Let the strip be inclined with the angle α to the screen at the distance z from its center. Then, the radius of curvature for the strip can be determined as

$$R = \frac{z}{\sin\alpha}. \quad (9)$$



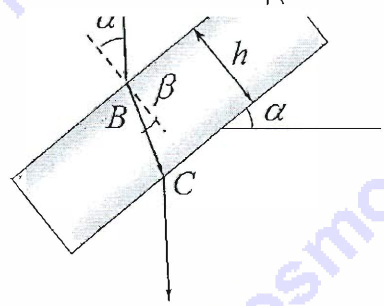
Consider the light transmission through the strip, inclined with the angle α. For this case, the optical length for the light is

$$l = |BC| = \frac{h}{\cos\beta} = \frac{h}{\sqrt{1-\frac{\sin^2\alpha}{n}}}, \quad (10)$$

where h denotes the strip thickness. For the derivation of equation (10) the refraction law $\sin\beta = \frac{\sin\alpha}{n}$ is used.

Consequently, the phase shift between the ordinary and extraordinary waves, which appear after the light transmission, is equal to

$$\Delta\varphi = \frac{2\pi}{\lambda} l\Delta n = \frac{2\pi}{\lambda}\frac{h}{\sqrt{1-\frac{\sin^2\alpha}{n}}}\Delta n = \frac{\Delta\varphi_0}{\sqrt{1-\frac{\sin^2\alpha}{n}}}, \quad (8)$$



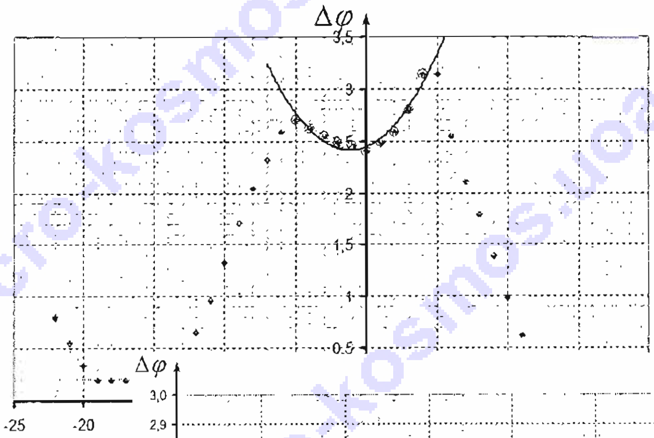
where $\Delta\varphi_0 = \frac{2\pi}{\lambda} h\Delta n$ stands for the phase shift for the uncurved strip, or the phase shift in the center.

At small angles α equation (8) can be simplified to

$$\Delta\varphi = \frac{\Delta\varphi_0}{\sqrt{1-\frac{\sin^2\alpha}{n}}} \approx \Delta\varphi_0\left(1 + \frac{z^2}{2n^2R^2}\right). \quad (9)$$

Thus, for determination of the radius of curvature, one needs to analyze the phase shift as a function of the square distance to the center of the strip.

Graph of the phase shift is drawn in the figure on right. Because we are interested in the central part of the graph, "reflection" parts are not shown. (This graph is not required from participants)



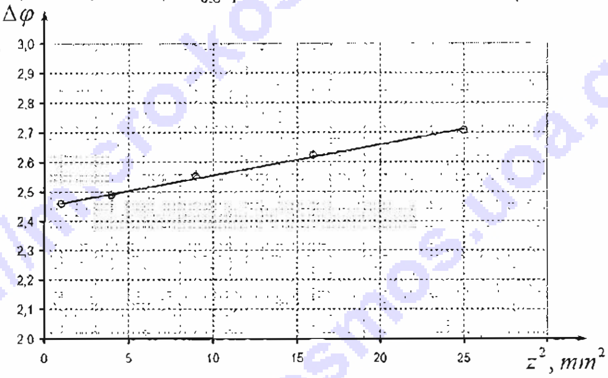
The graph shows that the central part is approximately parabolic function of z

$$\Delta\varphi = az^2 + b. \qquad (10)$$

In order to determine the coefficients of the function we can draw graph of $\Delta\varphi$ as a function of z^2 (see figure on the right). Using MLS, we can determine the parameters

$a = 0.0104$ мм$^{-1}$,

$b = 2.45$.



It is necessary to add 10π to the obtained value of b.

Comparing equations (9) and (10), we conclude that the parameters can be represented by the strip characteristics as:

$$\alpha = \frac{\Delta\varphi_0}{2n^2R^2}, \quad b = \Delta\varphi_0. \qquad (11)$$

From those equations we get the radius of curvature of the strip

$$R = \frac{1}{n}\sqrt{\frac{b}{2a}}. \qquad (12)$$

Substitution of the obtained results leads us to $R = 29\ mm$. Note that the obtained result is quite rough, due to uncertainties in measuring.