Parallel Dipole Line Magnetic Trap for Earthquake & Volcanic Sensing (10 points)

A. BASIC CHARACTERISTICS OF PDL TRAP

1. Determination of the magnet's magnetization (*M*) (2.5 pts)

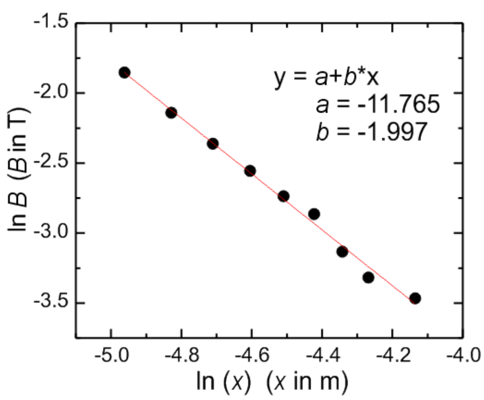
<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>A.1
0.1 pts</td><td>Record zero offset (B_0) of the Teslameter without any magnet nearby. Subtract subsequent field measurement with this value

Example from a Teslameter unit: B_0 = 0.86 mT</td><td>0.08 pts range (-10 mT to 10 mT)

Correct unit: 0.02 pts</td></tr>
<tr><td>A.2
1.15 pts</td><td>Measure magnetic field B vs. x in the near field region ($7 \le x \le 16$ mm). <u>Where x is the position measured from the center of the magnet</u>. Record and plot your result on the answer sheet.

x_0 = 4 mm, B_0=0.86 mT. Δx is measured from surface. B = Braw - B_0

<table>
<tr><th>Δx (mm)</th><th>X (mm)</th><th>Braw (T)</th><th>B (T)</th><th>ln(x) x in m</th><th>ln(B) B in T</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>3</td><td>7</td><td>0.1576</td><td>0.1567</td><td>-4.962</td><td>-1.853</td></tr>
<tr><td>4</td><td>8</td><td>0.1186</td><td>0.1177</td><td>-4.828</td><td>-2.139</td></tr>
<tr><td>5</td><td>9</td><td>0.0951</td><td>0.0942</td><td>-4.710</td><td>-2.362</td></tr>
<tr><td>6</td><td>10</td><td>0.0785</td><td>0.0776</td><td>-4.605</td><td>-2.556</td></tr>
<tr><td>7</td><td>11</td><td>0.0657</td><td>0.0648</td><td>-4.510</td><td>-2.736</td></tr>
<tr><td>8</td><td>12</td><td>0.0579</td><td>0.0570</td><td>-4.423</td><td>-2.864</td></tr>
<tr><td>9</td><td>13</td><td>0.0445</td><td>0.0436</td><td>-4.343</td><td>-3.132</td></tr>
<tr><td>10</td><td>14</td><td>0.0371</td><td>0.0362</td><td>-4.269</td><td>-3.318</td></tr>
<tr><td>12</td><td>16</td><td>0.0321</td><td>0.0312</td><td>-4.135</td><td>-3.466</td></tr>
</table>

Plot:


</td><td>Correct label and unit for data: 0.1 pts

Number of correct data for x <= 16 mm:
0.05 pts for each correct data, max 0.45 pts

Plot:
-Correct axis label and unit: 0.05 pts

- Using around 75% of plot area: 0.05 pts

-For each correct data point: 0.05 pts, max. 0.4 pts

-Adding trendline: 0.1 pts</td></tr>
</table>

A.3 0.75 pts	*Use your experimental data to determine the value of the exponent p.* Linear regression (LR) $y = a + b\,x$: $B = \frac{\mu_0 m}{2\pi\ L} \frac{1}{x^p}$ $\ln(B) = a - p\ln x$ where $a = \ln\left(\frac{\mu_0 m}{2\pi L}\right)$. LR yields : *a* = *-11.765* and *b* = *-1.997* The power exponent: *p* = *-b* = *2.0* Note that this is in very good agreement with the exact result: at short distance ($x < L$) a diametric (or a dipole line) magnet has $B \sim 1/r^2$ dependence. See Ref. [1] , Fig. 2c.	Obtaining *p* from graph: 0.05 pts Obtaining *p* from linear regression: 0.1 pts Result: *p* = 1.8 - 2.2 : 0.65 pts *p* = 1.6 - 2.4 : 0.35 pts Result with wrong sign: *p* = (-1.8) – (-2.2) : 0.4pts *p* = (-1.6) – (-2.4) :0.1pts More than two sig. figs.: minus 0.05 pts
A.4 0.5 pts	*Determine the magnet's magnetization M.* $m = \frac{2\pi L}{\mu_0}\exp(a) = 0.987\,\text{Am}^2$ $M = \frac{m}{\pi R^2 L} = 1.2\text{x}10^6\,\text{A/m}$ This is close to the more accurate results from more extensive measurements to far field (see Ref. [1], Fig. 2c) and *we use this value for subsequent questions:* $M = 1.1\text{x}10^6\,\text{A/m}$	Correct unit: 0.05 pts Obtaining intercept (*a*) from graph: 0.025 pts Obtaining intercept from LR: 0.05 pts Correct formula for *m* and/or *M* : 0.1 pts Result for *M* (x10^6 A/m): 0.9 - 1.4 : 0.3 pts 0.1 - 2.5 : 0.15 pts More than 2 sig. figs.: minus 0.05 pts

2. The Magnetic Levitation Effect and Magnetic Susceptibility (χ) (1 pts)

Question	Answer	Marks
A.5 0.1 pts	*Place gently a graphite rod HB/0.5 and length = 8 mm. Measure the levitation height* y_0 *of the rod (see Fig. 7a). Hint: Use the insert ruler provided as shown in Fig. 7b. Press the ruler on the magnets to read the position of the graphite rod* We levitate graphite HB/0.5, l = 8mm. Using the insert-ruler, we measure approximately Δy = 1 mm from the top of the magnet surface. Thus: $y_0 = R - \Delta y$ = (3.2 - 1) mm = 2.2 mm	correct unit: 0.02 y_0 = (1.7 - 2.2) mm: 0.08 pts partial credit: Only Δy = (1 - 1.5) mm: 0.03 pts

A.6 0.8 pts	*Use the result from part A.5 to determine the magnetic susceptibility χ of the graphite rod.* Solving for χ: $mg = F_y = -\frac{\mu_0 M^2 \chi V_R}{2} \frac{R^4}{a^5} f_Y(y_0/a)$ $\chi = -\frac{2\rho g a^5}{\mu_0 M^2 R^4 f_Y(y_0/a)}$ We calculate: $a = R + g_M/2$ =(3.2+1.5/2) mm=3.95 mm. Using y_0 = 2.2 mm: $f_Y(u) = \frac{4u(3-u^2)(1-u^2)}{(1+u^2)^5}$, $f_Y(y_0/a) = f_Y(2.2/3.95)$=1.07 Using the correct $M = 1.1\text{x}10^6$A/m; and R = 3.2 mm, ρ = 1680 kg/m^3 we have: $\chi = -1.85\text{x}10^{-4}$. Note that this is very good agreement with the literature value for graphite pencil lead: $\chi = -2\times10^{-4}$ (see Ref.[1], pg. 2 & Ref.[2]). The sign is negative indicating a diamagnetic material.	Correct expression for χ: 0.4 pts Result for χ (x10^{-4}) -(1.4 to 2.6) : 0.4 pts -(0.5 to 4) : 0.2 pts Wrong sign: minus 0.1 pts
A.7 0.1 pts	*What kind of magnetic material is graphite? Choose one: (i) Ferromagnetic; (ii) Paramagnetic; or (iii) Diamagnetic?* (iii) Diamagnetic. Because: (1) Graphite is repelled by magnetic field (2) The sign of χ is negative.	Correct choice: 0.1 pts

3. The camelback potential oscillation and magnetic susceptibility (χ) (1 points)

Question	Answer	Marks
A.8 0.2 pts	*Perform an oscillation for the "HB/0.5" graphite and l =8 mm. Limit to small oscillation amplitude i.e. A < 4mm. Determine the oscillation period. (The oscillation will decay over time due to damping, ignore this damping effect).* Example, we measured 5 oscillations of HB/0.5 with length l = 8 mm. We displaced it by ~3 mm and let it oscillates. We measured 5 oscillation periods:	Correct label and unit: 0.02 pts Number of correct data each 0.01 pts, max 0.03 pts Number of oscillation < 3 : 0 pts >= 3 : 0.05 pts T_z = (1.2 - 1.5) s: 0.1 pts

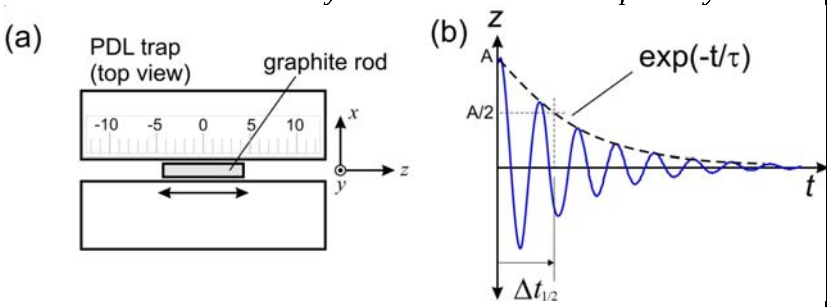
Trial	5 Tz (s)				
1	6.12				

2	6.13				
3	6.14				

Average : T_z = 1.23 s

A.9 0.8 pts	*Calculate the magnetic susceptibility (χ) of the graphite using this oscillation* For harmonic oscillator : $k_z = m_R\,\omega^2$, solving for χ: $$\chi = -\frac{k_z}{C_1\mu_0 M^2 V_r} = \frac{\omega^2\rho}{C_1\mu_0 M^2}$$ Using the correct $M = 1.1\text{x}10^6$A/m. Using C_1 = 198.6/m^2, and T_z = 1.23 s, we obtain $\chi = -1.5\times10^{-4}$. Note that this is in good agreement with the literature value of the graphite pencil lead: $\chi = -2\times10^{-4}$ (Ref.[1], pg. 2); and the sign is negative indicating a diamagnetic material.	Correct expression for χ: 0.4 pts Result for χ (x10^{-4}) -(1.4 to 2.6) : 0.4 pts -(0.5 to 4) : 0.2 pts Wrong sign: minus 0.1 pts

4. Oscillator quality factor (Q) and estimate of air viscosity μ_A (3.0 points)

Question	Answer	Marks
A.10 0.5 pts	*We need to determine the damping time constant of the oscillation τ. Sketch how you measure τ in a simple way .*  The trick is to use "half-time" concept of exponential decay. We set the oscillation and measure the time taken for the amplitude to halve. The lifetime is: $$\tau = \frac{\Delta t_{1/2}}{\ln 2}$$	Correct idea: 0.3 pts Correct expression for τ: 0.2 pts
A.11 1.5 pts	*Perform oscillation damping experiments with a group of rods with various diameters and fixed length of 8 mm. Determine the damping time constant τ for each rods*	Correct label and unit 0.1 Number of correct data

We displaced the graphite by ~4 mm, started the stopwatch and then waited until it decays to half.

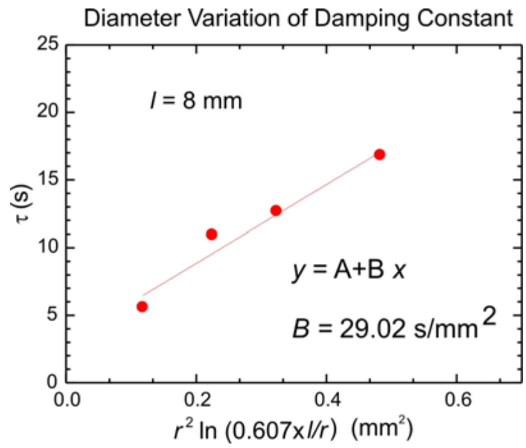
Trial	Diam.	Actual Radius	$\Delta t_{1/2}$	Mean $\Delta t_{1/2}$	τ	r^2xln(0.607 l/r)
	(mm)	(mm)	(s)	(s)	(s)	(mm^2)
1	0.3	0.19	3.89	3.913	5.646	0.117
			3.97			
			3.88			
2	0.5	0.28	7.69	7.617	10.989	0.224
			7.57			
			7.59			
3	0.7	0.35	8.77	8.82	12.73	0.322
			8.81			
			8.88			
4	0.9	0.45	12.4	11.70	16.88	0.482
			11.33			
			11.38			

for each diameter (4):
< 3 : 0.1 pts
>=3 : 0.25 pts
(max 1.0 pts)

Positive monotonic trend for τ vs. diameter from 0.3 to 0.9 mm with τ = 5 to 20 sec : 0.4 pts

A.12
1 pts

Determine the air viscosity μ_A



We have: $\tau = b r^2 \ln\left(0.607 \times \frac{l}{r}\right)$, where: $b = \frac{2}{3}\frac{\rho}{\mu_A}$. We performed linear regression $y = a + b\,x$, with $y = \tau$ and $x = r^2 \ln\left(0.607 \times \frac{l}{r}\right)$. We obtain: b = 29.02 s/mm^2.

$$\mu_A = \frac{2}{3}\frac{\rho}{b} = 38.6\ 10^{-6}\ \text{Pa.s} \quad (1\ \text{Pa.s} = 1\ \text{kg /m s})$$

Note that this is about 2.1x the actual viscosity of air of 18.2μ.Pa.s. The discrepancy is due to the ellipsoidal

Correct unit: 0.05

Obtaining result with linear regression or plot: 0.25 pts

Result μ_A (x10^{-6} Pa.s):
20 - 60 : 0.7 pts
10 - 80 : 0.4 pts
1 - 100 : 0.1 pts

	approximation of the Stokes drag (vs. the actual cylindrical shape of the rod) and the proximity effect of the rod to the magnet (wall effect). Another factor is the crude nature of our manual τ determination. See Ref. [1], pg. 8.	

B. SENSOR APPLICATION OF THE PDL TRAP

5. PDL Trap Seismometer (0.5 pts)

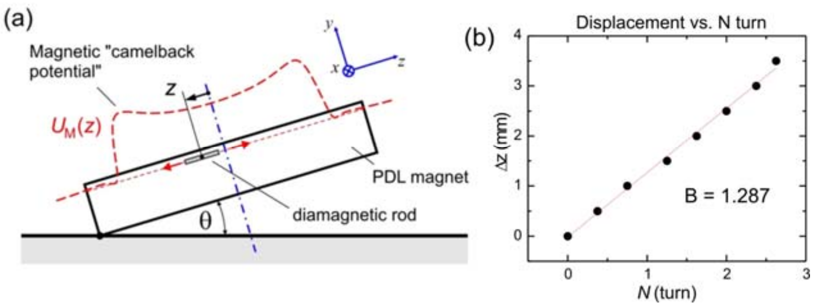
Question	Answer	Marks
B.1 0.2 pts	*Which diameter of rod do you choose?* To obtain the lowest acceleration noise floor "a_n" we should choose the largest diameter graphite i.e. 0.9 mm, because their damping time is the longest and the mass is the largest.	Correct answer: 0.2 pts
B.2 0.3 pts	*Calculate the seismometer acceleration noise floor (a_n) for the rod of your choice!* For HB/0.9 and length l = 8 mm: We use τ = 16.9 s; and T = 298 K, we have: $m_R = \rho \pi r^2 l$ = 8.55x10^{-6} kg : $$a_n = \sqrt{\frac{4k_B T \omega_0}{Q m_R}} = \sqrt{\frac{8k_B T}{\tau m_R}} = 1.5\text{x}10^{-8} m/(s^2 Hz^{0.5})$$	Correct unit: 0.1 Correct answer: 0.2 pts

6. PDL Trap Tiltmeter (2 pts)

Question	Answer	Marks
B.3 0.5 pts	*Derive the relation theoretically between displacement Δz with the screw thread size S and the number of turns (N).* $$k_z \Delta z = m g \sin\theta = m g N S / D \qquad \Delta z = \frac{m g S N}{k_z D}$$ From Question 3, we also have $k_z = m\omega^2$: $$\Delta z = \frac{g S}{\omega^2 D} N$$	Correct expression: 0.5 pts Partial credit $k_z \Delta z = m g \sin\theta$: 0.2
B.4 1.25 pts	*By turning the screw slowly, determine the rod displacement Δz vs. the number of screw turns (N). Determine the thread size S*	Correct label and unit: 0.1 pts

We measured the distance between screws: D = 22 cm, and we used the period from Q3: $T_z = 1.23$ s

Δz (mm)	ϕ	N (turn)			
0	0	0			
0.5	135	0.375			
1	270	0.75			
1.5	450	1.25			
2	585	1.625			
2.5	720	2.0			
3	855	2.375			
3.5	945	2.625			



By performing linear regression: $y = a + b\,x$

We have b = 1.287 mm/turns : $S = \dfrac{b\omega^2 D}{g}$ = 0.75 mm/turn.

This is reasonably close to the actual value of the thread size: S = (0.8±0.1) mm/turn.

Distance between screws: $22.8 < D < 22.2$ cm : 0.1 pts

Number of correct data:
< 3 sets : 0 pts
3-5 sets: 0.15 pts
>5 sets : 0.25 pts

Obtaining result with linear regression or plot: 0.2 pts

Result:
$0.7 < S < 0.9$: 0.55 pts
$0.5 < S < 1.1$: 0.15 pts

Correct unit for S : 0.05

B.5 0.25 pts

When the ground tilt changes we want the graphite rod to go to equilibrium as fast as possible (instead of sustaining very long oscillation) to allow easy reading. What is the ideal Q factor for a tiltmeter?

We need critical damping thus: $Q = 0.5$

Correct Q : 0.25 pts

REFERENCES:

[1] Gunawan, O. & Virgus, Y. *The one-dimensional camelback potential in the parallel dipole line trap: Stability conditions and finite size effect.* J. Appl. Phys. 121, 133902, (2017). DOI:10.1063/1.4978876.

[2] Gunawan, O., Virgus, Y. & Fai Tai, K. *A parallel dipole line system.* Appl. Phys. Lett. 106, 062407, (2015). DOI: 10.1063/1.4907931.