Wiedemann-Franz Law – Solution

Part A: Electrical conductivity of metals (1.5 points)

A.1 (1.0 points)

Magnet descend time:

Number	Copper [s]	Aluminum[s]	Brass [s]
1	17.77	9.23	6.1
2	17.96	9.39	5.83
3	18.16	9.22	6.04
4	18.15	9.37	5.86
5	17.76	9.36	6.16
6	18.2	9.44	5.92
7	17.67	9.65	5.9
8	17.9	9.18	6.08
9	17.67	9.41	5.86
10	18.36	8.96	5.99
Average	17.96	9.32	5.97

A.2 (0.5 points)

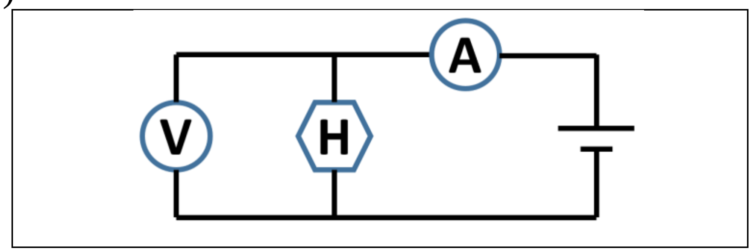
	Copper	Aluminum	Brass
Electrical conductivity $\left[\frac{1}{\Omega m}\right]$	5.97×10^7	2.98×10^7	1.60×10^7

Part B: Thermal conductivity of copper (3.0 points)

B.1 (0.1 points)

Rod 1 temperature : **22.76 [C]**

B.2 (0.5 points)



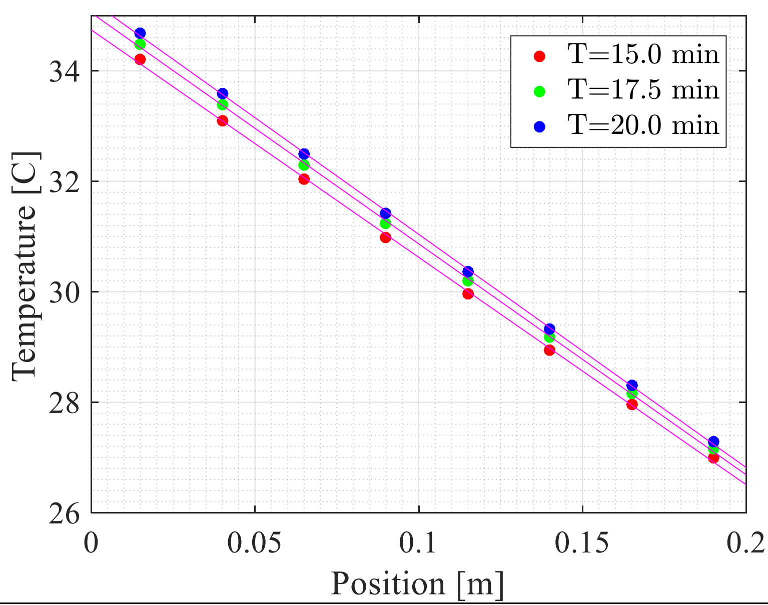
B.3 (0.1 points)

$P = I \cdot V = 5.51[\mathrm{W}]$

B.4 (0.5 points)

Time [S]	T1 [C]	T2 [C]	T3 [C]	T4[C]	T5 [C]	T6 [C]	T7 [C]	T8 [C]
900	26.98	27.96	28.95	29.96	30.98	32.03	33.10	34.20
1050	27.16	28.16	29.17	30.20	31.240	32.30	33.38	34.48
1200	27.29	28.30	29.33	30.37	31.42	32.49	33.58	34.68

B.5 (1.0 points)



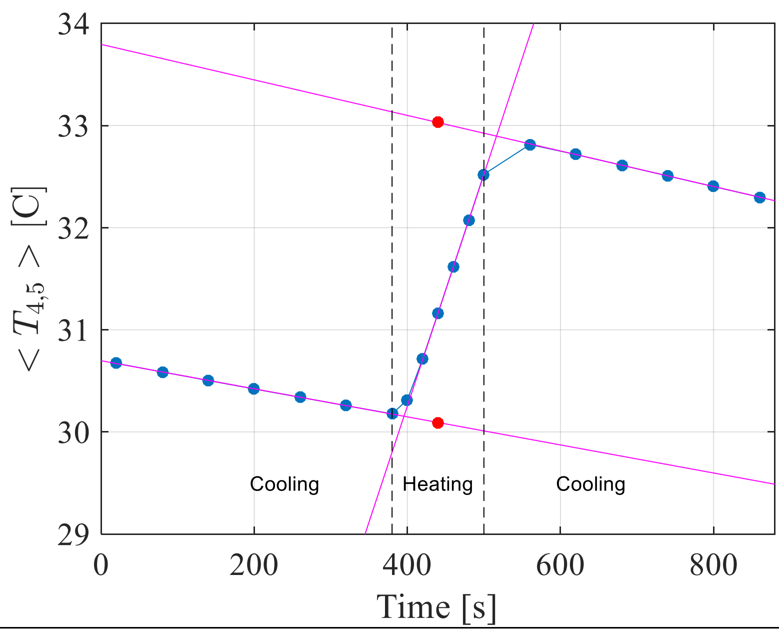
B.6 (0.5 points)
$\kappa_0 = -\frac{P}{A\frac{\Delta T}{\Delta x}} = -\frac{5.51[W]}{\pi\cdot\left(10^{-2}[m]\right)^2\cdot\left(-41.8\left[\frac{K}{m}\right]\right)} = 420\left[\frac{W}{mK}\right]$ $\frac{\Delta T}{\Delta t} = \frac{31.04[C]-30.62[C]}{5\cdot 60[s]} = 1.4\cdot 10^{-3}\left[\frac{K}{s}\right]$
B.7 (0.3 points)   **<u>higher value</u>**   We expect a **<u>higher value</u>** of κ_0 compared with the real κ_{cu} because of 2 reasons: 1. A part of the supplied heat power is lost through the side walls. Therefore, the heat transfer through the cross-section of the rod is smaller.   2. Since the system is not in a steady state ($\frac{\Delta T}{\Delta t} \neq 0$), the corresponding power involved should be subtracted from the power supplied by the heater.

Part C: Heat loss and heat capacity of copper (4.0 points)

C.1 (1.0 points)

$Time[s]$	$T_1[C]$	$T_2[C]$	$T_3[C]$	$T_4[C]$	$T_5[C]$	$T_6[C]$	$T_7[C]$	$T_8[C]$	$T_{av}[C]$
20				30.67	30.67				30.67
80				30.59	30.59				30.59
140				30.50	30.50				30.50
200				30.42	30.42				30.42
260				30.34	30.34				30.34
320				30.26	30.26				30.26
380				30.18	30.18				30.18
400				30.38	30.25				30.31
420				30.87	30.56				30.72
440				31.37	30.96				31.16
460				31.85	31.38				31.61
480				32.32	31.82				32.07
500				32.78	32.26				32.52
560				32.88	32.75				32.81
620				32.73	32.70				32.72
680				32.61	32.61				32.61
740				32.51	32.51				32.51
800				32.40	32.40				32.40
860				32.30	32.30				32.30

C.2 (1.0 points)



C.3 (1.0 points)

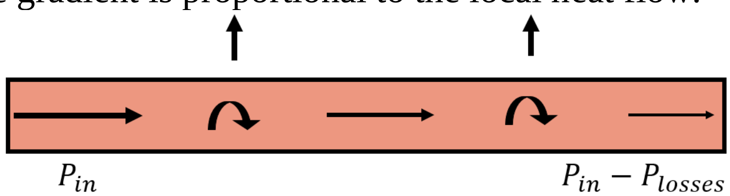
The purpose of this part is to correct to first order the result in part B. Hence, every solution within 10% accuracy is accepted (see marking scheme).

Solution 1 (using slopes):	Solution 2 (using jump):
$P_{loss} = c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ $P_{in} = c_p \cdot m \cdot \left(\left.\frac{\partial T_{av}}{\partial t}\right\|_{Heating} - \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling} \right)$ Where $\left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ is the average of both cooling slopes. $c_p \cdot m = \frac{5.5[W]}{\left(2.27 \cdot 10^{-2}\left[\frac{K}{s}\right] + 1.6 \cdot 10^{-3}\left[\frac{K}{s}\right]\right)}$	$P_{loss} = c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ $P_{in} \cdot \Delta t = c_p \cdot m \cdot \Delta T$ Where $\left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ is the average of the two cooling slopes, and ΔT is the extrapolated jump in temperature half way though the heating time interval. $c_p \cdot m = \frac{P_{in} \cdot \Delta t}{\Delta T} = \frac{5.5[W] \cdot 120[s]}{2.94[K]} = 224\left[\frac{J}{K}\right]$

$c_p \cdot m = 226\left[\frac{J}{K}\right] \Rightarrow c_p = 390\left[\frac{J}{kg \cdot K}\right]$ Which is 1% off the correct value. $P_{loss} = 226\left[\frac{J}{K}\right] \cdot 1.4 \cdot 10^{-3}\left[\frac{K}{s}\right] = 0.32[W]$	$c_p = 386\left[\frac{J}{kg \cdot K}\right]$ which is the correct value. $P_{loss} = 224\left[\frac{J}{K}\right] \cdot 1.4 \cdot 10^{-3}\left[\frac{K}{s}\right] = 0.31[W]$

C.4 (1.0 points)

The temperature gradient is proportional to the local heat flow.



To first order, the average temperature gradient will be proportional to the average heat flow. Therefore, the temperature gradient will be proportional to $P_{in} - \frac{1}{2}P_{losses}$:

$$\kappa = \frac{P_{in} - \frac{1}{2}P_{absorb} - \frac{1}{2}P_{loss}}{A \cdot (\Delta T / \Delta x)} = \frac{P_{in} - \frac{1}{2}c_p \cdot m \cdot \frac{\Delta T}{\Delta t} - \frac{1}{2}\dot{Q}_{loss}}{A \cdot \Delta T / \Delta x} = \kappa_0 \cdot \frac{P_{in} - \frac{1}{2}c_p \cdot m \cdot \frac{\Delta T}{\Delta t} - \frac{1}{2}\dot{Q}_{loss}}{P}$$

$$\kappa = 420\left[\frac{W}{mK}\right] \cdot \frac{5.51[W] - \frac{1}{2} \cdot 226\left[\frac{J}{K}\right] \cdot 1.4 \cdot 10^{-3}\left[\frac{K}{s}\right] - \frac{1}{2} \cdot 0.32[W]}{5.51[W]} = 396\left[\frac{W}{mK}\right]$$

Which gives an error of 2.5% error compared to expected $385\left[\frac{W}{mK}\right]$. We expect a 1% systematic error (see appendix).

Part D: Thermal conductivity of multiple metals (1.0 points)

D.1 (0.1 points)

$T = 22.65[C]$

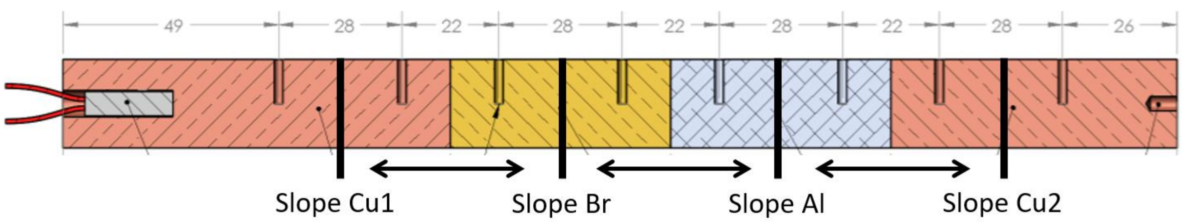
D.2 (0.2 points)

Time of measurement: $1041[s]$

$T_1[C]$	$T_2[C]$	$T_3[C]$	$T_4[C]$	$T_5[C]$	$T_6[C]$	$T_7[C]$	$T_8[C]$
41.68	40.51	38.51	34.65	32.47	30.71	29.63	28.62

$\Delta T_{cu1}/\Delta x$	$\Delta T_{Br}/\Delta x$	$\Delta T_{Al}/\Delta x$	$\Delta T_{cu2}/\Delta x$
$41.79\left[\frac{K}{m}\right]$	$137.86\left[\frac{K}{m}\right]$	$62.86\left[\frac{K}{m}\right]$	$36.07\left[\frac{K}{m}\right]$

D.3 (0.7 points)



$$\kappa_{Brass} = \kappa_{Copper} \cdot \frac{\frac{2}{3}(\Delta T_{cu1}/\Delta x)+\frac{1}{3}(\Delta T_{cu2}/\Delta x)}{\Delta T_{Br}/\Delta x} = 115\left[\frac{W}{mK}\right]$$

$$\kappa_{Aluminum} = \kappa_{Copper} \cdot \frac{\frac{1}{3}(\Delta T_{cu1}/\Delta x)+\frac{2}{3}(\Delta T_{cu2}/\Delta x)}{\Delta T_{Al}/\Delta x} = 239\left[\frac{W}{m\cdot K}\right]$$

Part E: The Wiedemann-Franz law (0.5 points)

E.1 (0.5 points)

	Copper	Aluminum	Brass
$\sigma\ [\Omega^{-1} m^{-1}]$ Electric conductivity	5.97×10^{7}	2.98×10^{7}	1.60×10^{7}
$\kappa \left[\frac{W}{Km}\right]$ Heat conductivity	396	239	115
$L \left[\frac{W\Omega}{K^2}\right]$ Lorenz coefficient	2.21×10^{-8}	2.67×10^{-8}	2.40×10^{-8}

Wiedemann-Franz Law – Marking Scheme

Part A: Electric conductivity of metals (1.5 points)

A.1	Measuring magnet fall (1.0 pts)	
	The number of total measurements : if $N \leq 15$	0.2 pts
	if $15 < N \leq 21$	0.5 pts
	if $N > 21$	0.7 pts
	Average travel time within 10% of solution for 2 out of 3 rods	0.3 pts
A.2	Calculation of conductivity (0.5 pts)	
	Correct calculation of conductivity from A1	0.1 pts
	Final result for 2 out of 3 values: Within 10% of correct value	0.4 pts
	Within 20% of correct value	0.2 pts

Part B: Thermal conductivity of copper (3.0 points)

B.1	Writing room temperature with units	0.1 pts
B.2	Design a 4-probe circuit (0.5 pts)	
	Drawing ammeter in series with source and heater	0.2 pts
	Measuring voltage on heater and not power source	0.3 pts
B.3	Writing the equation for power and proper calculation	0.1 pts
B.4	Writing thermometers readings (0.5 pts)	
	Complete set (24 temperatures in table)	0.2 pts
	Units	0.1 pts
	2 digits after decimal point	0.1 pts
	Times within 1 minute of requirement (15,17.5,20 minutes)	0.1 pts
B.5	Thermal equilibrium graph (1.0 pts)	
	All 24 points are plotted	0.4 pts
	Correct axes, with units	0.2 pts

	Points span on 1/2 the area of graph paper	0.2 pts
	Slope is sketched for 17.5 min	0.2 pts
B.6	Obtaining κ_0 (0.5 points)	
	Correct expression for κ_0	0.1 pts
Op.1	Range of κ_0 $\left[W/(mK)\right]$: $404 \le \kappa_0 \le 446$	0.2 pts
	$382 \le \kappa_0 \le 468$	0.1 pts
	Range of $\Delta T/\Delta t\left[K/s\right]$: $1.25\cdot10^{-3} \le \Delta T/\Delta t \le 1.55\cdot10^{-3}$	0.2 pts
	$1.1\cdot10^{-3} \le \Delta T/\Delta t \le 1.7\cdot10^{-3}$	0.1 pts
Op.2	The value of the corrected κ (using the method in the solution) with κ_0 ,$\Delta T/\Delta t$ and c_p, P_{loss} from the official solution is in range:	
	$376 \le \kappa \le 416$	0.4
	$356 < \kappa < 376$ or $416 < \kappa < 436$	0.2
B.7	Correct answer - **Higher value**	0.3 pts

Part C: Heat loss and heat capacity of copper (4.0 points)

C.1	Cooling-Heating-Cooling cycle (1.0 pts)	
	Number of measurement points for each step: if $3 \le N < 5$	0.1 pts
	if $N \ge 5$	0.2 pts
	Heating step time in range $1[\text{min}] \le t \le 3[\text{min}]$	0.2 pts
	Cooling steps time $t > 200[s]$	0.2 pts
	If average between T4,T5 or average over all thermometers	0.2 pts
	Used only T4 or only T5	0.1 pts
	The reported temperature mid-heating is:	
	Less than 2.5 [C] away from average temperature in B.4	0.2 pts
	Between 2.5[C] and 4.0[C] from average temperature in B.4	0.1 pts

C.2	Cooling – Heating – Cooling graph (1.0 pts)	
	Correct axes, units on axes	0.2 pts
	Number of points on graph: $N \geq 15$	0.4 pts
	$12 \leq N < 15$	0.2 pts
	Points span on 1/2 the area of graph paper	0.2 pts
	Slope lines are plotted for cooling steps	0.2 pts
C.3	Obtaining c_p and P_{loss} (1.0 pts)	
	$P_{loss} = c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$	0.2 pts
	$P_{in} = c_p \cdot m \cdot \left(\left.\frac{\partial T_{av}}{\partial t}\right\|_{Heating} - \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling} \right)$ or $P_{in} \cdot \Delta t = c_p \cdot m \cdot \Delta T$	0.4 pts
	Range of c_p in $\left[J/(kg \cdot K) \right]$: $425 \leq c_p \leq 350$	0.2 pts
	$465 \leq c_p \leq 310$	0.1 pts
	Range of P_{loss} in $[W]$: $0.25 \leq P_{loss} \leq 0.38$	0.2 pts
	$0.19 \leq P_{loss} \leq 0.44$	0.1 pts
C.4	Correct κ (1.0 pts)	
	$c_p \cdot m \cdot \frac{\Delta T}{\Delta t}$	0.1 pts
	$c_p \cdot m \cdot \frac{\Delta T}{\Delta t}$ and P_{loss} are treated the same way	0.1 pts
	Form of equation $\kappa = \frac{\kappa_0}{P}\left(P - \alpha \cdot \left(c_p \cdot m \cdot \frac{\Delta T}{\Delta t} + P_{loss}\right)\right)$	0.2 pts
	Writing that $\alpha = 0.5$	0.3 pts
	κ range in $\left[W/(mK) \right]$: $376 \leq \kappa \leq 416$	0.3 pts
	$356 < \kappa < 376$ or $416 < \kappa < 436$	0.2 pts

Part D: Thermal conductivity of multiple metals (1.0 points)

D.1	Writing temperature with units	0.1 pts
D.2	Temperature measurements (0.2 pts)	
	Measurement time is greater than 15 minutes	0.1 pts
	Correct calculation of $\Delta T / \Delta x$ using $28mm$ spacing	0.1 pts
D.3	Calculation of κ for other metals (0.7 pts)	
	general form of $\kappa_\alpha = \kappa_{copper} \cdot \frac{Slope}{(\Delta T / \Delta x)_\alpha}$	0.1 pts
	Weighted average: 1:2 and 2:1 average between coppers (correct direction, see solution)	0.4 pts
	Weighted average but wrong weights	0.2 pts
	Slope from closest copper or simple average	0.1 pts
	$103[W/(mK)] \le \kappa_{brass} \le 126[W/(mK)]$	0.1 pts
	$215[W/(mK)] \le \kappa_{Aluminum} \le 263[W/(mK)]$	0.1 pts

Part E: The Wiedemann-Franz law (0.5 points)

E.1	Wiedemann-Franz law table (0.5 pts)	
	Calculation of Lorenz number, using absolute temperature	0.1 pts
	$2.12[W\Omega/K^2] \le L_{copper} \le 2.39[W\Omega/K^2]$	0.2 pts
	$2.13[W\Omega/K^2] \le L_{Brass} \le 2.71[W\Omega/K^2]$	0.1 pts
	$2.00[W\Omega/K^2] \le L_{Aluminum} \le 2.54[W\Omega/K^2]$	0.1 pts

Please note that this marking scheme might change, particularly the ranges.