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An Educational MONTE CARLO Simulation/Animation Program for the Cosmic Rays Muons and a Prototype Computer-Driven Hardware Display

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An educational software for the study and the detection methods of the cosmic rays muons, passing through several light transparent materials (water, air, etc.), is described. Muons are created and Cherenkov photons are produced. Their paths and interactions are simulated and visualised/animated on the computer screen using MONTE CARLO methods/techniques (which employ random numbers following certain distributions). Finally the photons are detected. The simulated detector used is a Cherenkov counter, which is an array of photodetectors. The photodetectors are presented on the computer screen or/and by a prototype computer-driven hardware display, which has been/can be developed by students using simple electronics.

From the educational point of view, there is a widely accepted perspective of teaching physics triggered by everyday life phenomena, using for their explanation the "inside" view and the study of microcosm. According to this, microcosm processes such as the cosmic rays have penetrated in all grades of physics education. The subject of cosmic rays is a very important phenomenon, because it happens every second of everybody's life, since almost 1 particle/cm^2/sec penetrates our bodies. So it is very important that every student, at every grade (all the people) gets to know this subject.

Microcosm processes cannot be observed with the naked eye, so the only way to observe the cosmic rays is through computer simulation or/and their detection. Such simulation programs and detection devices may be very common for the researchers and an everyday experience to the experimentalists, but rarely exists in an educational science laboratory in a completed way, combining microscopic processes with macroscopic phenomena.

However, if the study of such phenomena and microscopic processes requires the use of the simulation abilities of the computer, we don't believe that we may, simply, visualise these processes in an approximate, artistic way based mainly on our imagination, but instead, we have to "reproduce" these processes in the computer according to the physics laws. The most effective and realistic way to achieve this reproduction is to use MONTE CARLO methods/techniques (Kalkanis, 1996), which use random numbers to feed a relevant formulation (based on the physics laws to which physical processes are obeying) and the randomness (with what the real data are produced). MONTE CARLO (M.C.) programs are very common to physics theoreticians and experimentalists, especially to those working on high-energy physics. Educational M.C. programs may be also very useful, especially for reproducing (calculating and visualising) microcosm random processes.

The Cosmic Rays Muons (C.R.M.) program, or Cosmic Rays Library (CO.RA.L.),[1] is an educational simulation software package that, using MONTE CARLO routines, "creates" cosmic rays muons, "produces" Cherenkov photons from the interactions of the muons with the matter, calculates their paths and light cones, and "detects" their "hits" on hypothetical photomultiplier arrays, shown on the computer screen. Furthermore and supplementary, these hits are activating the corresponding light emitting diodes (L.E.D.s) of a prototype computer-driven hardware display that simulates the photomultiplier array. The package is accompanied by a database/help with cosmic information, organised in such a way that users of different levels of knowledge or interest or age have access to that, at the appropriate depth. Its demonstration to students of all grades (university and secondary, even elementary, school students), as well as to pre- and inservice training teachers, has given encouraging results and has triggered enthusiastic comments. It has to be noticed that, apart from being—hopefully—user-friendly, easy to run and to understand, and attractive, the program is scientifically correct and provides, in its most deeper levels, all relevant knowledge and formulas on the subject to the fond of learning users.

COSMIC RAYS MUONS AND THEIR DETECTION

The subject of "cosmic rays" (Hillas, 1972) implies that particles, mainly very energetic with an extraterrestrial or even extragalactical origin, penetrate the galaxy and react in the Earth's atmosphere producing mainly muons (hard leptonic component) (Christopoulou-Mavromichalaki, 1993). Most of the other components (pions) decay in flight to muons and neutrinos (which cannot be easily detected). The energy distribution of the cosmic rays muons consists of two exponential parts with different slides, which reveals two different origins of the cosmic rays: the galactic and the extragalactic:

$$J\left(E > E_i\right) = k_i E^{-\gamma_i}$$

where J is the cosmic rays flux with energy E greater than E_i , i=2 the number of slopes and E_i, k_i ,γ_i constants different for every exponential part (Hughes & Wu, 1977). When reproducing a C.R.M., this must have an energy that fits to the specific energy distribution. As it will be explained later this could be done only using MONTE CARLO techniques.

Muons that pass through matter lose energy due to several electromagnetic processes (Halzen & Martin, 1984; Perkins, 1982; Tait, 1980). The most important process (mainly for 1-2000 GeV muons) is the ionisation process (Longmann, Kopp, & Voss, 1985). When a muon passes through matter it ionises/excites the constituent atoms (the Cherenkov effect is due to this process).

The next important process is the process of radiation loss or, as it is called, the (muon) bremsstrahlung, due to the deceleration of the muons from the electric field of the atomic nuclei and atomic electrons of the matter (radiative "collisions"), and it appears in the form of photons. This process becomes the dominant process for muons with energies higher than 2000 GeV (Longmann, Kopp, & Voss, 1985).

Muons also lose energy due to direct electron pair production (according to the Quantum Electro-Dynamic (QED) theory), as well as to muon pair production (this contribution is very small). There is also a small contribution of nuclear interaction as a real photonuclear scattering. These interactions occur at the neighbour of nuclei (Longmann, Kopp, & Voss, 1985).

When a high-energy charged particle, as most of C.R.M.s are, traverse a light transparent medium (such as water or atmospheric air), it excites their atoms which emit photons (Cherenkov effect) forming cones (Cherenkov light cones). More specifically, if the velocity of the particle exceeds the velocity of the light in the dielectric, then, when its atoms de-excite, the emitted light appears in the form of a coherent wavefront at a fixed angle with respect to the trajectory. The cosine of this angle is inversely proportional to the velocity of the particle and of the refractive index of the medium (Zrelov, 1970), so from this angle the velocity of the particle could be calculated. The detection of the light/photons, and, if needed, the reconstruction of the particle path, is usually achieved by photomultiplier arrays. This is a common technique in large high-energy or cosmic rays detection experimental sites (The DELPHI Collaboration, 1983, 1984, 1991; The DELPHI Barrel RICH Group, 1991, 1992; Seguinot & Ypsilantis, 1994; Kalkanis, 1984; The NESTOR Collaboration, 1992a, 1992b, 1993; Kalkanis, Resvanis, & Voulgaris, 1992; Eichler & Learned, 1981; Aprille et al., 1988; Ayres et al., 1985; Battistoni et al., 1985) (see also Appendices).

THE MONTE CARLO SIMULATION SOFTWARE

To simulate the cosmic rays behaviour and their detection, their main components (muons) must be produced with the same momentum and trajectory characteristics as those of the cosmic rays. Every muon produced must fulfill the differential energy distribution:

$$J(E > E_i) = k_i E^{-\gamma_i} \ , \ E_i \le E < E_{i+1} \quad \Rightarrow \quad dJ/dE = -k_i \gamma_i E^{-\gamma_i}$$

As we have mentioned, the MONTE CARLO methods is a way to reproduce quantities that follow a certain distribution (James, 1980). To reproduce a quantity that follows a uniform distribution is quite easy since computers produce pseudo-random uniform numbers. For all the other distributions different techniques have been developed. This is exactly the case of the cosmic rays exponential energy distribution. We follow the mathematical method that uses the inverse cumulative distribution function (Salicio, 1985) in order to generate from random uniform distribution available by the computers a non uniform distribution. The cumulative distribution function of E is

$$G(E) = \int_{E_i}^{E} dJ(E')/dE' \ dE'$$

which equals the probability of finding in the energy range of E_i and E. The value y=G(E) is uniformly distributed. To produce the desired energy distribution, we need to derive the inverse function $E=G^{-1}(y)$:

$$y = G(E) = C(E_i) - k_i E^{-\gamma_i}, E_i \le E < E_{i+1} \text{ where } C(E_i) = k_i E_i^{-\gamma_i},$$

so

$$E = \left[\left(C(E_i) - y\right)/k_i\right]^{-\frac{1}{\gamma_i}} \qquad C(E_i) \le y < C(E_{i+1})$$

The trajectory of the muons is randomly determined. At a plane above the edge of the detector, an (x,y) point is chosen using two uniform random numbers: x and y. The polar angles j_1 and j_2 are also uniformly distributed. The value of φ_1 is a uniform random number from 0° to 180°, and φ_2 is also a uniform random number, in the range -90° to 90°.

During the muon's travel in the medium, its path is divided into small time steps. At every step the program calculates the energy loss of the muon, using the formulas that give the energy loss of muons due: to ionisation, to the Bremsstrauhlung effect, to the Direct electron and muon pair production and to the nuclear interaction. (The formulas of the energy loss are in differential form [Longmann, Kopp, & Voss, 1985]. The program integrates them and makes a mathematical co-processor necessary to decrease dramatically the run-time.) When the muon travels with velocity that exceeds the velocity of light in the medium, it emits photons forming the surface of a cone about its trajectory as axis, at every step of the muon's movement. Using the formulas of the Cerencov radiation (Perkins, 1982), the number of photons as well as the angle from the trajectory are extracted and distributed on a cone of the given angle (Cherenkov angle). It is supposed that the photons are uniformly distributed on the surface of the cone. To take into account the conservation of momentum, one quarter of the photons are distributed uniformly at the first quadrant, producing a uniform distribution of the angle *w* from 0° to 90° and for every photon three more symmetric to the first, at the second (180°-w), the third (180°+w), and the fourth (-w) quadrant.

Every photon that travels through matter could be attenuated due to the processes of photoelectric absorption, the Compton scattering and the pair production. The possibility of a photon not to be attenuated is $e^{-x/xo}$, where x_0 is the attenuation length (medium dependent) and *x* the length of the trajectory of the photon until it reaches a photodetector (if any). After defining this photodetector, taking into account its distance, the probability $e^{-x/xo}$ is calculated. This probability is uniformly distributed. To decide for a single photon, if it will manage to reach a photodetector, we use a uniform random number. If this is between 0 and $e^{-x/xo}$, the photon reaches the photodetector, else, it will be attenuated.

MONTE CARLO is also simulating the behaviour of the different components that integrate a detector. The photodetectors, from which the lattice-detector is made of, have fluctuations in their behaviour an acceptance of about 10%. To decide exactly which photon is detected, a uniform random number (from 0 to 1) is used. If it is smaller that 0.1 the specific photon is detected: otherwise it isn't. As it is easily noticed, MONTE CARLO techniques are applied to every step of the process/program.

METHODOLOGY

The subject of cosmic rays is very difficult to be taught (difficult in the sense that students/teachers rarely have access to experimental areas and the phenomenon is not visible with naked eye). The traditional way of teaching such a subject (and all the microcosm subjects) is through formulas and drawings, either on a blackboard or on a computer screen according to a teacher's or programmer's imagination. Our approach is a MONTE CARLO simulation program that informs/teaches in an interactive way, reproduces the reality, and presents it on the computer screen in a dynamic way or/and through a hardware display, which, actually, can be constructed by the students themselves.

More analytically, three main educational purposes are intended. The first is of course to give as much detailed information as possible about the cosmic rays and their detection, beginning from the physical aspect of view and then to proceed to a more mathematical approach through formulas and diagrams. The second is actually to see an "event": a simulated detection of a cosmic ray muon that passes through a detector array. The last and maybe the most important one (which corresponds actually to a new approach) is to "produce" cosmic rays particles defined by the user who gives their characteristics and may, also, change (if he/she wants) the characteristics of the detector (i.e. the length, the acceptance of the photodetector, even their geometry from cubic to cylindrical), as well as the physical parameters (the attenuation length of the medium, turn on and off the energy loss processes, etc.) in order to really understand on what does detection of the cosmic rays depends.

Since cosmic rays are an everyday phenomenon, this educational software must apply to every student at every educational level as well as to the teachers, even though it has been actually developed for university students. At the elementary level, the teacher may show several pictures from the database as well as from the detection simulation to the students, adding his/her own comments. At the secondary level, the students can use by themselves the program displaying some of the already produced events on the screen (e.g., changing the view angles) or/and on the computer-driven hardware display. The university students can additionally produce their own events, changing several physical constants, the material, the shape and the characteristics of the detector, and so forth, and watch by themselves how the results are changed on the screen or/and on the computer-driven hardware display.

Concerning the use of the program by university students (and consequent teachers), who may need not only a phenomenological view of the cosmic rays aspect but, also, the theoretical and mathematical ones, the simulation used has to be based (and is based, indeed,) on MONTE CARLO techniques which assures that the "reproduction" and visualisation of the events are accurate and according to the current knowledge of the corresponding physical processes and laws. On the other hand, students at lower levels who must get to know cosmic rays only phenomenologically, although they must use the same program and techniques with the same accuracy and credibility (instead of another "simplified" and less correct model), are properly instructed (both by the program and the teacher) to follow the suitable navigation in order to have access only to those that are understandable to them.

FEATURES OF THE PROGRAM

The structure of the CO.RA.L. program is made according to the abovementioned purposes. The program is divided into four modules:

The first module is the so-called "Database," which contains all the information about the cosmic rays and their detection. Our decision to have the same database for all levels of education forced us to the following construction: The database is divided into pages and levels (horizontal and vertical structure). Each page contains information, formulas, and diagrams. The user can change from page to page (using buttons) and from level to level (using the hypertext technique, keywords, and "key-images"). Some of the database images are not simple pictures but are created simulating the phenomenon. For example, to demonstrate the transformation of primary cosmic rays to secondary ones, the angle of the produced particles, their energy, their attenuation, and the interaction ratio are calculated using the MONTE CARLO technique (Figure 1). Additionally, muons lose energy due to several interactions. To understand the nature of this energy loss, one has to understand separately each interaction process. To achieve this, the user can "turn on and off" interactions and watch how they contribute to the total energy loss. Generally dealing with such composite phenomena, the user must not only have the ability to change the parameters but also to understand the model itself by changing its component processes (Andaloro, Donzelli, & Sperandeo-Mineo, 1991).

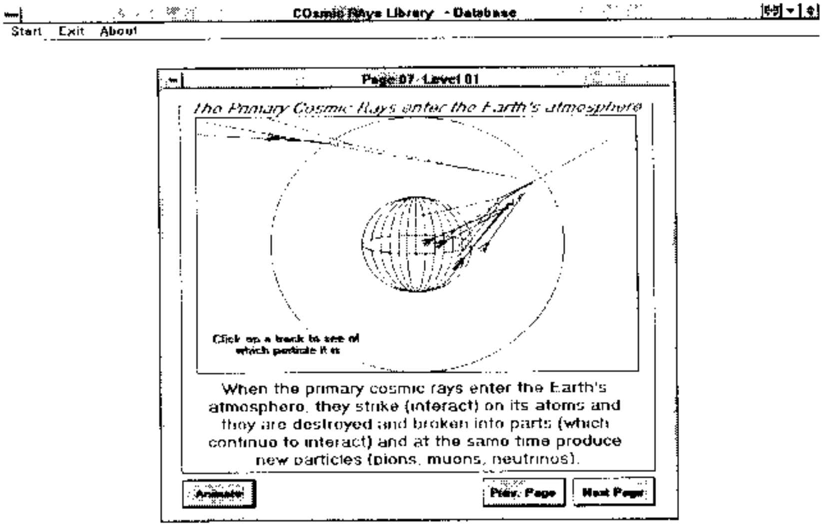

Figure 1. Simulation of the transformation of primary to secondary cosmic rays

The second module is the so-called "Parameters Determination," where the user willing to produce a new event can define the characteristics of the detector, physical parameters of the medium, and so forth. Every question is accompanied by a brief and simple description of what exactly is asked and if necessary where this will be used by the program. When the user is about to ask a question the descriptions must be automatically presented in the same form (visible) explaining how and where they affect the program running (self-explanatory). Also there is a default value which the user can accept it, especially the first-time user (Depireux, Jodl, & Wilson, 1988). All questions are grouped into forms, so the student visualises the interference of different parameters and how they influence the program through a user-friendly interface.

The third module is the so-called "Event Production"; here the program simulates the detection of a muon, using the parameters already defined at the second module and the MONTE CARLO techniques that had been described.

The fourth module is the so-called "Visualisation/Animation," which shows on the screen (or/and on a prototype computer-driven hardware display if it exists), an already produced event or an event produced by the user using the third module. The program draws in 3D, the lattice detector, the trajectory of the muon, and for several time steps (each time step is equal to the refresh time of the detector), it "lightens" the photodetectors that have been hit (Figures 2 and 3). At the same time, it draws the Cherenkov cones about the muon's track according to theory. After this image, it becomes clear to the user that the hits at the detector are provoked by the Cherenkov photons which are emitted on a cone about a muon's trajectory. The user can change the 3-plane angle from which he or she observes the event, or the user can observe it from the muon's system of coordinates.

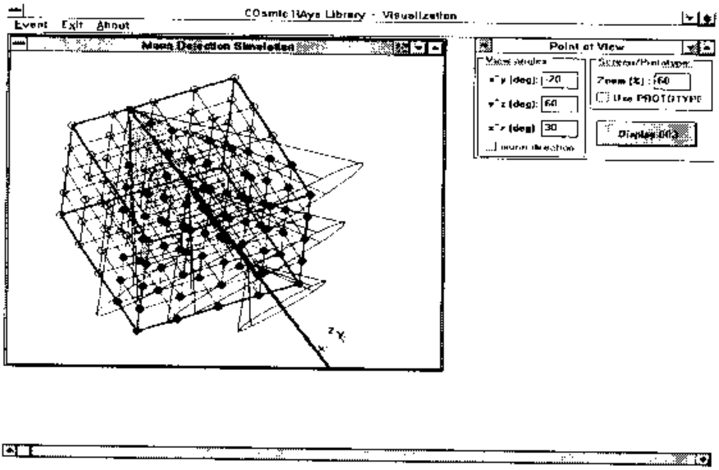

Figure 2. Cosmic rays muons detection

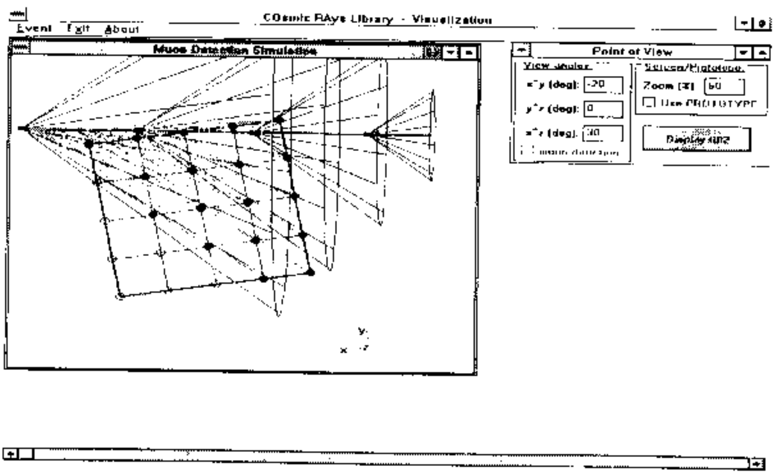

Figure 3. Photon hits at the photodetector array and the Cherenkov cones

PROTOTYPE DETECTOR DISPLAY

The idea is to use in addition to the computer screen, a hardware display to make more lively the way a Cherenkov cone is detected. A cubic lattice frame can be constructed (hopefully by the students) with L.E.D.s in the place of the photodetectors. When an event is visualised on the screen, at the same time, the L.E.D.s that correspond to the photodetectors that have been hit, turn on. This way the Cherenkov cone is clearly demonstrated, because it is shown in 3-D space and not through its 2-D projection only.

The cubic lattice frame can be constructed from plexiglass rods. At every junction an L.E.D. is placed to act as the display of the corresponding photodetector. For demonstrations and manipulation/transportation reasons, the prototype is advisable to have 5x5x5=125 computer-driven L.E.D.s, which corresponds to 125 photodetectors.

For educational reasons, this prototype is advisable to be constructed by the students (even high school students can construct it, as it has been proved by our tests). There is no need for any interface since only the three parallel ports of the IBM PC are needed (Cohen, 1992). This simplicity, of course, costs time: only one L.E.D. can be turned on at one time. In order to have "on" more than one L.E.D. - photodetectors "simultaneously", the L.E.D.s are turned on and off fast (persistence).

ASSESSMENT/EVALUATION

The program CO.RA.L. has been experimentally used and tested by (a) high school students and their teachers[2], (b) inservice training general and physics teachers[3], (c) university students/future general teachers[4], and (d) university students/future physics teachers[5]. Although some members of those experimental groups had previously attended some cosmic rays classes or courses, we asked all of them to fill out pre- and after-testing questionnaires, different of course for each group. In parallel an interview with the students/teachers was posted, helping us to interact with them and to obtain a more clear and quantitative impression of our program's effectivity.

High school students, knowing almost nothing about cosmic rays before the testing (since the subject is not included in their curriculum), seemed to gain profit from all visualised information found in the program. On the other hand, they didn't pay too much attention to formulas and diagrams. From the database, they studied carefully the simulation pages (for example the page that simulates the motion of a primary cosmic rays particle), and they watched different muon detection events from several view angles. After testing, they were able to answer questions concerning the origin and nature of the cosmic rays, the way they are produced and travelling in the air, as well as the way they are detected. They were astonished by the fact that millions of muons, invisible to the eye, are penetrating their bodies continuously. They were also impressed by the large distances these particles travel until they reach Earth. The most important for us is that they seemed enthusiastic running the program and learning things such as the nature of cosmic rays and their origin. It has to be noticed and emphasised that a small group of high school students constructed the hardware L.E.D. display, which is used for the whole demonstration.

University students/future general teachers and inservice training general teachers who previously attended some elementary courses about cosmic rays subject appeared to have a nebulous idea on cosmic rays, before running the program. They expressed their ideas on cosmic rays by obscure and short, laconic phrases, such as "strange particles (or radiation...) coming from the universe," or even "... a kind of radioactivity...." after running the program they were able to describe clearly 90% of the concepts and information presented by the program. Most importantly they seemed confident about informing others and explaining to them these concepts. Moreover, they were willing and anxious to use the program in order to extend their teaching to this everyday, but in general unknown subject, through its visualisation routines and the hardware display. Some of the teachers noticed that probably large distances and numbers met at astrophysics and microcosm observation which appeared together with small distances of microcosm simulation and microscopic processes, they could confuse the students. They have come to this conclusion because they always have in mind the possible use of the program by their own students and so they are mainly interested in the pedagogical aspects and aim of the program. Taking into account these remarks, the latest versions of CO.RA.L. explain these "strange" quantities in everyday terms and a comparison of them in a didactic way (Figure 4).

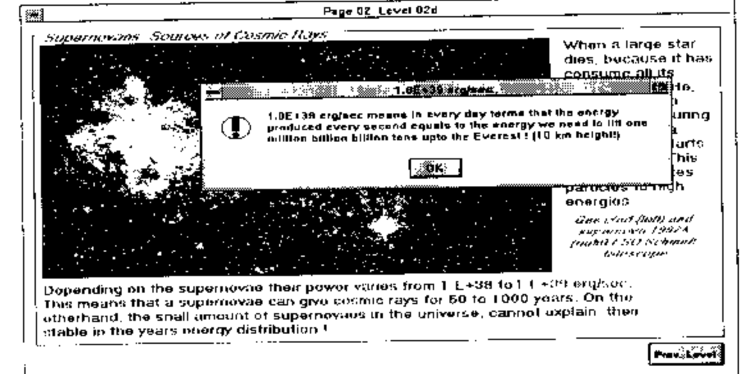

Figure 4. "Strange" numbers are explained in everyday terms

After running the program, university students/future physics teachers and inservice training physics teachers (who had taken special courses on cosmic rays) appeared to have clarified most of their preconcepts on the subject. For example they realised that a cosmic rays particle does not have a certain "signature," but its signal strongly depends on many statistical laws. They understood that, except from some very general signal characteristics, the events differ a lot and they realised how difficult and important it is to subtract the cosmic rays background that appears in all high-energy physics experiments.

Their interest on the subject also seemed to increase. They spent much of the time generating and displaying (on the screen or/and on the hardware apparatus) their own events. They were looking for more and different options for creating and observing cosmic muons. After their navigation to all options of the program, almost all of them noticed that they were not overwhelmed or overdosed by the program, as everybody is after a condensed traditional course. Instead, they noticed that it would be a great advantage for them if it could deal with more materials and more kinds of particles (such as neutrinos), as well as with more complex detector's geometry. This is of course part of our intentions for the next versions of CO.RA.L.

In general, the program has been characterised sufficiently as user-friendly and feasible. The users had strongly emphasised that the program was neither giving some artistic views, according to the physicists'/programmer's imagination, nor repeated, over and over again all the same, in a predefined, deterministic way.. According to their remarks, the users were impressed and enthusiastic as they realised that the MONTE CARLO methods give the most accurate description of the reality and not only for the microcosm processes but also for everyday phenomena that depends on a large number of parameters. We believe that these last characteristics of the program were the main reasons for the high effectiveness that proved to have the tested educational program among the students and the teachers.

CONCLUSIONS

This program, together with the prototype detector display, has been used by university students and high school students as well as by inservice training centres teachers with encouraging results. It has been characterised as user-friendly feasible, and that it covers most of the questions a student would ask about cosmic rays both in a qualitative and formalistic way. Also it presents in a very clear way how statistical laws influence the microcosm research.

All students and future or current teachers agreed that the prototype detector display, apart of making the lesson more attractive and interesting, it helps dramatically to visualise and understand the Cherenkov cone geometry because its 2-D projection on the screen is not quite understandable. Additionally, the students who constructed the prototype understood a lot of physics aspects (L.E.D.s, currents, and voltages), as well as computer aspects (the way computers send signals through their parallel port to the environment—robotics). It is noticed that students of almost every level (grade) can proceed to the construction of such a prototype computer-driven display.

In general, we think that educational software, making use of MONTE CARLO techniques, is a very effective way of teaching phenomena such as microcosm phenomena and their processes. Additionally the use of actuators connected to the computer in a simple and cheap way can help students understand the physics hidden in the way computers interchange signals with the environment (the principles of robotics).

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Notes

1. The set up disks of the program coral.zip is available by ~ftp/pub/philos/educat info: kalkanis@vxcern.cern.ch or/and mms@ellinogermaniki.gr
2. From two private high schools in Athens.
3. At the Peripheral In-Service Training Center of the Greek Ministry of Education at Piraeus.
4. In the Pedagogical Department P.E. of the University of Athens.
5. In the Physics Department of the University of Athens.

APPENDIX A: ENERGY LOSS OF MUONS PASSING THROUGH MATTER

High energy muons that pass through matter loose energy due to several electromagnetic processes: ionization, Bremsstrauhlung, direct electron apir production and nuclear interaction.

I. Ionization

When a charged particle passes in the neighbour of an atom it transfers a fraction of its energy to an (or more) electron of the atom. This electron escapes from the atom and so it is ionized. This phenomenon is called "ionization." This process is the dominant process for moderate relativistic velocities. The energy loss of a muon passing through matter due to ionization is given by the Bethe-Bloch formula:

$$\frac{dE}{dx} = \alpha^2 2\pi N\lambda_e^2 \frac{Zm_e}{A\beta^2}\left\{\ln\frac{2m_e\beta^2\gamma^2E'_m}{I^2(Z)} - 2\beta^2 + \frac{1}{4}\frac{E'^2_m}{E^2} - \delta\right\},$$

where α is the fine structure constant, N is the Avogadro's number, λ_e is the electron's Compton wavelength, Z and A are the atomic number and the atomic weight of the transverse medium, mμ and m_e are the rest masses of the muon and the electron that escapes from a medium's atom respectively, β=p/E where p is the muon's momentum, $\gamma=E/m_i$ where E is the muon's energy, I(Z) is the medium's mean ionization potential, E'_m is the maximum energy transferable to the electron and δ is the density correction.

As a conclusion, the ionization energy loss for a certain medium is proportional to α^2 and inversly proportional to β^2. This means that for moderate relativistic velocities (β) the ionization process is the dominant (order of α^2).

II. Bremsstrahlung

As we know all accelerated charges radiate electromagnetic waves, losing energy. This radiation travels in the form of a photon . This effect is called Bremsstrahlung radiation and the cross-section and the energy loss of muons due to this process is given by the following formulas:

$$\frac{d\sigma}{dv} = \alpha^3 Z(Z+1)\left(2\lambda_e \frac{m_e}{m_\mu}\right)^2 \frac{1}{v}\left(\frac{4}{3} - \frac{4}{3}v + v^2\right)\Phi(\delta),$$

$$\frac{dE}{dx} = E\frac{N}{A}\int_{v_{min}}^{v_{max}} v\frac{d\sigma}{dv}dv$$

where v is the fraction of muon's energy transferred to the photon, $\Phi(\delta)$ is a function of δ which is the minimum momentum transfer to the medium's nucleus. All the other variables have already been defined at the ionization process.

As we can see from the above formula the energy loss is inversly proportional to the mass of the charged particle (of the muon). So, particles of smaller mass radiate more dramatically. As a result, this effect is most significant for the electrons and secondly for the muons. Additionally, the energy loss is proportional to the muon's energy and α^3.

At moderate relativistic energies the Bremstrahlung contribution (of order α^3) is smaller that the ionization's contribution (of order α^2), but as the muon energy increases the contribution of Bremstrahlung increases while the one of the ionization decreases dramatically. Generally for energetic muons found in the cosmic rays, radiative effects dominate.

III. Direct electron pair production

Some times the energy that the muon radiates produces an electron pair. This process is called direct electron pair producation and it is another radiative process. The cross section and the energy loss of this process are given by the following formulas:

$$\frac{d^2\sigma}{dv d\rho} = \alpha^4 \frac{2}{3\pi} Z(Z+1)\lambda_c^2 \frac{1-v}{v}\left(\Phi_e + \frac{m_e^2}{m_\mu^2}\Phi_\mu\right),$$

$$\frac{dE}{dx} = 2E\frac{N}{A}\int_{v_{min}}^{v_{max}} v \int_0^{\rho_{max}} \frac{d^2\sigma}{dv d\rho} d\rho dv$$

where ρ is the asymmetry parameter of the electron-positron pair, and the terms Φ_e and $\Phi\mu$ correspond to different QED diagrams with atomic and nuclear form factors corrections.

IV. Nuclear Interaction

High-energy muons also gives another kind of interaction, the photonuclear interaction. In this interaction the energy loss produces a hadronic shower. The photonuclear interaction cross-section and energy loss are given by the following formulas:

$$\frac{d\sigma}{dv} = \frac{\alpha}{2\pi} A\sigma_{\gamma N} v \left\{ \frac{3}{4} G(x) \left[\kappa \ln\left(1 + \frac{m_1^2}{t}\right) - \frac{\kappa m_1^2}{m_1^2 + t} - \frac{2m_\mu^2}{t} \right] + \frac{1}{4} \left[\kappa \ln\left(1 + \frac{m_2^2}{t}\right) - \frac{2m_\mu^2}{t} \right] + \frac{m_\mu^2}{2t} \left[\frac{3}{4} G(x) \frac{m_1^2}{m_1^2 + t} + \frac{1}{4} \frac{m_2^2}{t} \ln\left(1 + \frac{t}{m_2^2}\right) \right] \right\}$$

$$\frac{dE}{dx} = E \frac{N}{A} \int_{v_{min}}^{v_{max}} v \frac{d\sigma}{dv} dv$$

where G(x) is a function of the total nuclear cross-section, which is also a function of the photon energy using the latest models which are in agreement with the cosmic rays data. All the other variables have the same meanings as in the previous sections. As we can see the energy loss is proportional to the Energy of the muon but its contribution is quite small and it becomes measurable only at very high energy muons.

V. Direct Muon Pair Production

The photon's energy that a muon radiates could be produce a muon pair. The cross section of this process is negligible and we have not take it into account, because it doesn't exceed 0.01% of the total energy loss for any material.

APPENDIX B: CERENKOV RADIATION

When the muon travels with velocity that exceeds the velocity of light in the medium, it emits photons forming the surface of a cone about its trajectory as axis. This radiation is called Cerenkov radiation. The half angle of the Cerenkov cone ϑ_c as well as the number of emitted photons N are given by the following formulas.

$$\vartheta_C = \arccos(1/n\beta) \approx \sqrt{2(1 - 1/n\beta)}$$

where n is the index of refraction and β=u/c the muon's velocity. Additionally,

$$N = \frac{\alpha}{c}\int\left(1 - \frac{1}{\beta^2 n^2}\right)2\pi d\nu \approx 500\sin^2\vartheta_C\ /\ cm\ (visible\ spectrum)$$

which is the number of emitted photons per cm of path length. The Cerenkov angle and the number of photon increase as the velocity of the particle increases.