

Teaching Transverse (but not Longitudinal?) ElectroMagnetic Waves – The Underlying Reasons and an Instructional Approach

N. Sarris¹, H. Londos¹, G. Kalkanis²

¹Solid State Section of Physics Department

²Science, Technology and Environment Laboratory of Pedagogical Department P.E.
University of Athens

Abstract

The electromagnetic wave equations, derived from Maxwell's equations, impose that these waves are exclusively transverse, a fact that is experimentally verified in vacuum or in homogeneous and isotropic media, whilst mechanical waves propagate in matter either in transverse or in longitudinal mode. This arises numerous questions and appears to generate epistemological obstacles to university students. A study investigating the underlying reasons that prohibit the propagating of longitudinal electromagnetic waves, lead to the structure and properties of space-time, according to the special theory of relativity, which covariantly transforms Maxwell's equations. Then, an instructional approach, considering mechanical and electromagnetic wave models as totally independent conceptual systems, applied and evaluated to physics major students and physics teachers, seems to overcome these obstacles, whilst it manifest the unified image of natural world.

Background, Aims and Framework

The universal wave equation describes all types of waves, both transverse and longitudinal, as those propagated in matter; the mechanical waves. The corresponding equations for the plane electromagnetic (e-m) waves, derived from Maxwell's eqs, imply that these waves are exclusively transverse, a fact that is experimentally verified in vacuum or in homogeneous and isotropic media. In the literature, the subject is treated in a rather fragmentary way, which is not of any educational value. Consequently, most textbooks (representatively [1-4]) are content with this statement in a very analytical way, whilst they do not comment further –if not at all– on the non-propagation of longitudinal e-m waves. When teaching e-m waves to university students, numerous questions arise about the underlying reasons for this absence. A research among physics major students has shown that a complete picture is still lacking even in postgraduate level. An easy answer that refers to the fundamental character of Maxwell's eqs induces a much more nebulous picture. Furthermore, student's questions recall the case of mechanical waves, which are propagated both in transverse and longitudinal mode, expecting apparently an analogous for e-m waves. Although this searching for analogies in physics indicates a positive attitude and effort to constitute a unified model / worldview, it also reveals some serious misconceptions that appear to be the result of their pre-/in-university traditional instruction. The students, having taken courses on mechanical waves earlier than those on e-m waves, have built a conceptual framework, which induces a "mechanical" approach, a "metaphor" in any effort to understand the formation and propagation of e-m waves. They appear to overlap or mix-up the conceptual frameworks of mechanical and e-m waves, forming epistemological obstacles to the acquisition of scientifically correct knowledge.

Thus, a study to investigate the underlying reasons for the non-propagation of longitudinal e-m waves was undertaken. And, then, profited by the student's triggered interest, an instructional approach was designed, applied and evaluated, aiming to overcome these obstacles by means of the formation of independent conceptual

systems, whilst at the same time, relate these systems in such a way as to manifest the unified image of natural world, an image accepted by contemporary science.

In this framework, the representative theoretical models of the two phenomena, mechanical and e-m waves, are considered and introduced / studied as two totally independent conceptual systems [10]. This consideration does not permit any explanation or metaphor / analogy between the two different systems, thereby is leading to a complete distinction between them, imposing a precisely determined meaning on the concepts. It is assumed that any juxtaposition of the two paradigms' concepts should generate a "crisis", as the historical one, allowing to reveal any conceptual mixing. The basic concepts of each system, functioning as the nodes of the relevant conceptual network, acquire their meaning as parts of the whole system.

Mode of Inquiry

The undertaken study was coherent with the epistemological and cognitive approaches of the described framework. Mechanical waves and e-m waves were studied independently, in the context of classical Galilean principle of relativity (invariance of acceleration) and the special theory of relativity respectively, as well as in different media (vacuum, homogeneous and non-homogeneous, isotropic and non-isotropic). The instructional approach designed according to the same framework, was applied to over 300 physics major university students (experimental group) and evaluated in comparison with traditional instruction that was attended by an equal amount of students (control group), from 1998 to 2000. The research also included 60 in-service physics teachers of secondary education, during their training at the university, from 1999 to 2001. Both students and teachers attended courses relevant either to the traditional or the proposed approach, and then were interviewed.

Outcomes

Investigating –and, then, instructing– the propagation or not propagation of the longitudinal e-m waves (after a thorough and separate study of the mechanical waves), what was considered, first, was the effect of the Galilean relativity (invariance of acceleration / force) to Maxwell's eqs. In classical physics, Maxwell's eqs were assumed to hold exclusively in the ether system [4], accepting only transverse waves as a solution. However, it comes out that a transverse wave moving in ether along the x-axis ($\mathbf{k} = k\mathbf{e}_1$, $\mathbf{E} = E\mathbf{e}_2$, $\mathbf{B} = E\mathbf{e}_3/c$) has a longitudinal component of \mathbf{B}' in an inertial reference frame moving with velocity $\mathbf{u} = u\mathbf{e}_3$ with respect to ether, since: $\mathbf{B}' \cdot \mathbf{k}' = \mathbf{B} \cdot (-u + ck/k)/(u^2 + c^2)^{1/2} = -Eu/(c(u^2 + c^2)^{1/2}) \neq 0$ where \mathbf{k}' points to the direction of wave propagation in the primed system (making use of the classical Galilean law for the addition of velocities). It means that the classical Galilean relativity does not prohibit longitudinal e-m waves. On the other hand, as it is well known, the form of the Maxwell's eqs remains invariant under Lorentz transformation [5-7]. Lorentz transformation in turn links space and time in a unified structure according to the laws of special relativity. This means that the form of Maxwell's eqs is related with the space-time structure. Therefore it becomes therefore apparent that the structure of space-time, where the laws of special relativity hold, underlays the prohibition of longitudinal e-m waves.

Investigating –and, then, instructing– the propagation of longitudinal e-m waves in vacuum and in different media, put into consideration a non-dissipative linear homogeneous anisotropic medium (where $D_i = \epsilon_{ij}E_j$ and $\sigma = 0$), which exhibits an isotropic magnetic permeability (let $\mathbf{B} = \mu\mathbf{H}$, as in the case of ionic anisotropic crystals).

For plane e-m waves, ($\exp(i(k \cdot x - \omega t))$), the latest two equations combined with the Maxwell's eqs lead to two different polarizations (\pm) that satisfy two different dispersion relations (like the ordinary and the extraordinary rays in birefringence). Along the x-axis, it comes out that the wave number k_x is related to the frequency ω , as: $\alpha_{\pm} = k_x^2 / \mu\omega^2 = (\beta_{22} + \beta_{33} \pm ((\beta_{22} - \beta_{33})^2 + 4\beta_{23}^2)^{1/2}) / 2$, where $\beta_{ij} = \epsilon_{ij} - \epsilon_{i1}\epsilon_{j1}/\epsilon_{11}$

The corresponding e-m field amplitudes for (+) and (-) polarizations, are respectively:

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \gamma_+ \\ -\beta_{23} \\ \beta_{22} - \alpha_+ \end{pmatrix} \zeta, \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \frac{k_x}{\mu\omega} \begin{pmatrix} 0 \\ \alpha_+ - \beta_{22} \\ -\beta_{23} \end{pmatrix} \zeta, \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \gamma_- \\ \beta_{33} - \alpha_- \\ -\beta_{23} \end{pmatrix} \eta, \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \frac{k_x}{\mu\omega} \begin{pmatrix} 0 \\ \beta_{23} \\ \beta_{33} - \alpha_- \end{pmatrix} \eta$$

where $\gamma_+ = (\epsilon_{12}\beta_{23} - \epsilon_{13}(\beta_{22} - \alpha_+)) / \epsilon_{11}$, $\gamma_- = (\epsilon_{13}\beta_{23} - \epsilon_{12}(\beta_{33} - \alpha_-)) / \epsilon_{11}$; ζ and η measure the amplitudes (the electric field has a non-zero component along the wave vector [8]). Thus, although longitudinal e-m waves cannot propagate in vacuum, longitudinal components do exist in anisotropic media. On the other hand, in the case of e-m waves in an inhomogeneous medium (as in the case of a surface separating two different material media), for an arbitrary plane e-m wave incident on the surface, the refracted wave in the second medium is not transverse in general [9], exhibiting that inhomogeneity is another factor that leads to longitudinal e-m waves.

Conclusions and Implications

The investigation of the underlying reasons for the non-propagation of longitudinal e-m waves in vacuum and in homogeneous and isotropic media (and the propagation in inhomogeneous and anisotropic media), are related with (a) the homogeneous and isotropic media properties, which indicates that space is homogeneous and isotropic, (b) the space-time interconnection through special theory of relativity laws.

Incorporating those fundamental reasons to the theoretical model of e-m waves, the proposed instructional approach showed, according to the application / evaluation results, a significant feasibility and a remarkable efficiency (over 80% between the experimental and control groups) in overcoming misconceptions, as well as the constitution of a unified image of wave formation and propagation, as proposed from classical and neo-classical physics. This instruction approach has been proved also successful in the case of teaching quantum mechanics [11].

Bibliography

- [1] J.N. Shire, R.L. Weber. *Similarities in Physics*, Hilger Ltd, Bristol, 83, 1982;
- [2] J.R. Reitz, F.J. Millford. *Foundations of Electromagnetic Theory*. Addison-Wesley, NY, 300, 1975;
- [3] J.D. Jackson. *Classical Electrodynamics*. Wiley Sons, NY, 204, 1962;
- [4] W.K.H. Panofsky, M. Philips. *Classical Electricity and Magnetism*. Addison-Wesley, NY, 188, 1964;
- [5] D. H. Kobe. Derivation of maxwell's eqs from the local gauge invariance of QM. *Am. J. Phys.*, 46:342-348, 1978;
- [6] L.D. Landau, E.M. Lifshitz. *Classical Theory of Fields*. Pergamon, Oxford, 67-69, 1989;
- [7] W. Rindler. *Intr. to Special Relativity*. Clarendon, Oxford, 114-7, 1987;
- [8] L.D. Landau, E.M. Lifshitz, L.P. Pitaevskii. *Electrodynamics of Continuous Media*. Pergamon, Oxford, 334, 1993;
- [9] P. Halevi. Plane e-m waves in material media: Are they transverse waves? *Am. J. Phys.*, 48:861-867, 1980;
- [10] D. Stavrou, P. Hadzidaki, G. Kalkanis. *An instructional model for a qualitative approach to QM concepts*, proc. 8th Earli Eur. Conf., Göteborg, 1999;
- [11] P. Hadzidaki, G. Kalkanis, D. Stavrou. Quantum mechanics: a systemic component of the modern physics paradigm, *Phys. Educ.*, 35(6):386-392, 2000.